

## Lecture 1

- Units
- Estimating/Significant Figures
- Powers of ten
- Dimensional Analysis

Cutnell+Johnson: sections 1.1 –1.3, Appendices A, B and D

### Units

In physics, we look at the world and try to measure it. By measure, I don't mean necessarily its length, but any interesting quantity. A useful distinction in physics is between a qualitative measurement and a quantitative one. A qualitative measurement is one where we try to describe in words what is happening: "The sky is falling". A quantitative measurement is one where we associate a number or numbers with some aspects of what is happening: "Mount Vesuvius is spewing out lava at a rate of 500 kg/sec" or "The lava is hitting my house at a temperature of 1000° Celsius". In physics we always strive for a quantitative measurement, but it is important to remember that you always need to understand what is going on qualitatively for the numbers to mean anything.

When you've measured something, a number is useless unless you specify what its *units* are. This is painfully obvious of course: you must say it's "110 miles to Washington" and not just 110 to Washington: otherwise a hapless Canadian might think you're talking about kilometers, my son trapped in the back seat might think it's 110 hours before we stop, etc. I said this is painfully obvious, but we loyal taxpayers just squandered several hundred million dollars because different groups of people building a Mars probe didn't bother to check whether the other group was using metric or English units. So this is our gift to the Martians. The plus side is that the Martians may then get the movie "Spinal Tap": not one but two of the best jokes in the movie revolve around units. (**Extra Credit:** Name them.)

Thus in this class, a number without the proper units is just wrong (or actually, not even wrong, it's meaningless).

In this class, we'll almost always use metric units. We'll often use **MKS**, units, which stands for **M**eter, **K**ilogram, **S**econd. The fancy but useless name for these is **SI** units, coming from the French "Système Internationale". The main reason metric units are so convenient is that it's easy to change units if the need arises. If you're dealing with large quantities, the prefix "kilo" means 1000, i.e. a kilogram is a thousand grams, a kilometer is a thousand meters. If you're dealing with small quantities, the prefix "centi" means 1/100: a centimeter is one-one hundredth a meter, or in other words, there are one hundred centimeters in a meter. There is a table of these prefixes in the book, you certainly don't need to memorize them (I've never used "Deka" in my life), but you should know those two (and also "milli" = 1/1000), and certainly be able to recognize and look up the others.

However, since I and many of you grew up here, we have absolutely no intuition into them: we still think in the traditional English units. In the front of the book is a conversion table. It is useful to be able to quickly translate the two (especially when you're off on your European architecture tours and need to figure out how much cheese to buy in the deli). Thus for **estimation purposes only** they are

$$1 \text{ meter} \approx 1 \text{ yard}$$

$$1 \text{ kilometer} \approx 1/2 \text{ mile}$$

$$1 \text{ liter} \approx 1 \text{ quart}$$

$$1 \text{ kilogram} \approx 2 \text{ pounds}$$

They aren't very accurate, as you can see by looking at the exact numbers. (**In fact:** As we'll learn next week, the last relation is strictly speaking nonsense, the two aren't even the same unit. The correct conversion is

$$1 \text{ Newton} \approx 1/4 \text{ pound}$$

But On planet Earth, the kilogram – pounds conversion is useful, however.

By estimation purposes only, I mean you don't present them to me as an answer, but they are useful for checking if an answer to a question is sensible. However, it's often very useful to use these approximations in your head to make sure that an answer you're giving makes sense (i.e. the quantitative answer is in line with your qualitative guess, for example you don't want to say that Mount Vesuvius is spewing out 1 gram/second of lava).

As we'll see, from the meter, the kilogram and the second, we'll be able to build up almost every unit we'll ever use. What do I mean by "build up"? Say you want to buy some land, and need to measure an area. In the English system, we often use the ridiculous unit of "acres". It is much more convenient to use a unit built up from the basic unit. A square of  $1m$  (henceforth we abbreviate meter by  $m$ ) on each side has an area of  $1m^2$ , or one meter-squared. Thus we can measure areas in  $m^2$ . Similarly, a cube measuring  $1m$  on each side has a volume of  $1m^3$ .

The fact that we build up units out of MKS doesn't mean that every time you give an answer in class it has to be written in terms of these three. We'll use many other names. For example, a metric unit in common use in this country now is the liter. This is because a  $m^3$  is a relatively huge volume, so a liter is defined as

$$1000 \text{ liters} = 1m^3.$$

Notice that without the units, the equation  $1000 = 1$  makes no sense.

### Converting between units:

To convert between units, "multiply by 1". Here's the idea. Say we have a relation

$$x = y.$$

Then

$$\frac{x}{y} = 1.$$

For example, if

$$1000 \text{ liters} = 1m^3$$

then

$$\frac{1000 \text{ liters}}{1m^3} = 1 = \frac{1m^3}{1000 \text{ liters}}.$$

So now we can convert between different units which are measuring the same kind of quantity. For example, how many  $m^3$  in your favorite 2 liter bottle of soda?

$$2 \text{ liters} \times \frac{1m^3}{1000 \text{ liters}} = \frac{2}{1000}m^3 = .002m^3$$

This is probably why the unit of liter was invented:  $.002m^3$  does not sound very impressive.

We can now do a more complicated problem: How many miles is 1500 meters?

First, estimate the answer: 5280 feet in a mile, a meter is about a yard, which is three feet. Thus 1500 meters is a little less than a mile, about  $1500 \times 3/5280$  miles  $\approx .9$  miles. Now let's do it more carefully. From the front of the book, there are 3.281 feet in a meter.

$$1500m \times \frac{3.281 \text{ feet}}{1m} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = .9321 \text{ miles}$$

It's *very* useful to cancel out units which appear in both the numerator and the denominator. The units not canceled are those remaining in your final answer.

## Significant Figures

The preceding problem is a useful way of illustrating significant figures. When doing a quantitative analysis, it is absolutely crucial to remember how many significant figures you have. In plain language, this means: how far should we trust the computation? First, consider the estimate. We found that  $1500m$  is about 1 mile. This answer is good only to one significant figure, the “1” in the one mile. It would be pointless (and deceptive) to give the answer in this case any more accurately. The reason is that we used the relation

$$3 \text{ feet} \approx 1m.$$

This formula is accurate only to one significant figure. The formula accurate to two significant figures is

$$3.3 \text{ feet} \approx 1m.$$

Notice I have not just truncated the formula, I’ve rounded: 3.281 is closer to 3.3 than to 3.2. The formula I used above has four significant figures:

$$3.281 \text{ feet} \approx 1m.$$

If you were to use the one-significant-figure formula on your calculator, you would get  $1500 \times 3/5280$  miles = .85227272727 miles. This number should not be given this accurately, because the “3” used is only good to one significant figure, so one should just round the answer up to .9. This relation is only good to one significant figure, so we should only keep one significant figure. In the more accurate answer I gave above, the formula is good to 4 significant figures, so that is why I gave the answer with four significant figures, instead of  $1500 \times 3.281/5280$  miles = .93210227273 miles. The numbers beyond .9321 miles are meaningless, because the relation between meters and feet we have used is not exact, but good only to four places. Writing the extra digits in your answer is deceptive because it implies that you have calculated the number to this accuracy.

**Problem** To one significant figure, how dense is steel?

The piece of steel weighs  $80 N$ , and has a volume of  $64 in^3$ . 80 Newtons is about 20 pounds, so the density of steel is

$$\frac{20 \text{ lbs}}{64 \text{ in}^3} \approx .3 \text{ lbs/in}^3$$

## Estimating

As I mentioned before, being able to estimate your answer correctly is extremely useful in physics: it is a great way of checking your results. It is also extremely useful in all manners of

life. An estimate is **not** a guess: it is a calculation where you need to know the answer to one significant figure or less: say a factor of two. For example, here's the first calculation (not just a conversion) we'll do in this class. This one is useful for an architect designing an apartment building.

**Problem:** What is the mass of a waterbed?

**Answer:** Let's say a double bed. That's about say 6 feet long, 5 feet wide, and one foot deep. Thus the volume is about  $30 \text{ feet}^3$ . Most of the weight of the waterbed is in the water, so we'll use the formula for density of water, which is

$$1g/cm^3.$$

It's in the front of your book, but this is the kind of formula you should be able to memorize. I've written it here to only one significant figure, but in fact it's exact: grams and centimeters are defined so that this number is  $1g/cm^3$  to very high accuracy. To do the computation, we need to convert the volume of water in the bed to  $cm^3$ . First note that  $6 \text{ feet} \approx 2m$ ,  $5 \text{ feet} \approx 2m$ ,  $1 \text{ foot} \approx .3m$ , so the volume of the bed is

$$2m \times 2m \times .3m \approx 1m^3.$$

To convert this to  $cm^3$ , we should multiply by 1 in this way:

$$1m^3 \times \left( \frac{100cm}{1m} \right)^3 = 1000000cm^3.$$

Note the cubing on the left-hand side. That's necessary to cancel the  $m^3$ , and why the right-hand side ends up having units of  $cm^3$ . Now we can estimate the weight of a waterbed:

$$1000000cm^3 \times 1 \frac{g}{cm^3} = 1000000g \times \frac{1kg}{1000g} = 1000kg$$

Thus a waterbed weighs about  $1000kg$ , or on planet Earth, this is about 2000 lbs, or a ton. There's really no point in computing this number more accurately: different beds are different sizes, and anyway, when you're designing a building you design it to be much stronger than necessary. It doesn't matter if the bed is .8 ton or 1.2 ton.

**Problem:**

Is steel more or less dense than water?

**Answer:** The steel block in class weighs  $80N$ .  $N$  is the MKS unit of weight, and is called the Newton. To convert this to a mass, use the fact that  $W = mg$ . Thus the block has mass

$$m = \frac{W}{g} = \frac{80N}{9.8m/s^2} = 8.2kg$$

The block has volume  $64 \text{ in}^3$ . In MKS units,

$$64 \text{ in}^3 = 64 \text{ in}^3 \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right)^3 = 1.05 \times 10^3 \text{ cm}^3$$

This means that the mass density is

$$\frac{8200 \text{ g}}{1050 \text{ cm}^3} = 7.8 \text{ g/cm}^3$$

So steel is about 8 times as dense as water.

Let me emphasize again that an estimate is not a guess, although people often make a guess seem more authoritative by giving it a number. Here's one particularly absurd example. Before the space shuttle crashed, some NASA people had "estimated" that the probability of a shuttle crash was about 1 in 100,000. So here's a

**Problem:** Assume a space shuttle takes off every single day (including Labor Day, yes we have class then). If a space shuttle crashes 1 out of every 100000 flights, how long between crashes?

**Answer:**

$$\frac{1 \text{ crash}}{100000 \text{ flights}} \times \frac{1 \text{ flight}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ year}} \approx \frac{1 \text{ crash}}{300 \text{ years}}$$

It's ridiculous to think a new piece of technology will fail only once every three centuries.

## Exponential notation

All those zeroes we've been writing get pretty annoying, and when dealing with number like a billion or a trillion get excruciating. Luckily, there is a very simple and convenient way of dealing with them. This is to use *exponential notation*. I'm sure you've seen this before, but here's a quick review. The expression  $10^x$  means to multiply 10 by itself  $x$  times. The  $x$  is called the "exponent". Thus

$$10^1 = 10, \quad 10^2 = 10 \times 10 = 100, \quad 10^3 = 10 \times 10 \times 10 = 1000.$$

Then a million =  $1000000 = 10^6$ , a billion is  $10^9$ , a trillion is  $10^{12}$ . There are two crucial properties of exponentials. The first is that

$$10^a \times 10^b = 10^{a+b}.$$

For example,

$$10 \times 100 = 10^1 \times 10^2 = 10^{1+2} = 10^3 = 1000$$

$$100 \times 10000 = 10^2 \times 10^4 = 10^{2+4} = 10^6 = 1000000$$

This property lets us define negative exponents, i.e. when  $a$  or  $b$  is less than zero. Let's multiply

$$10^2 \times 10^{-1} = 10^{2+(-1)} = 10^1 = 10$$

Notice this is the same as

$$10^2 \times \frac{1}{10} = 10$$

Thus  $10^{-a}$  means that you multiply  $1/10$  by itself  $a$  times. This definition requires that  $10^0 = 1$ , for example

$$1 = 100 \times \frac{1}{100} = 10^2 \times 10^{-2} = 10^{2+(-2)} = 10^0.$$

The second important property of exponentials is that

$$(10^a)^b = 10^{ab}.$$

You can get this simply from the first property. For example

$$(10^a)^2 = 10^a \times 10^a = 10^{a+a} = 10^{2a}$$

We could have used these to do the waterbed computation without touching a calculator. The volume of the waterbed was  $1m^3$ . Since  $1m = 100cm = 10^2cm$ , in the volume  $1m^3 = (10^2cm)^3 = 10^{2 \times 3}cm^3 = 10^6cm^3$ . Then use the fact that  $10^3g = 1kg$  to get the final answer.

It's also easy to keep track of the number of significant figures with exponential notation. Any number can be written as a number between 1 and 10 times  $10^a$  for some exponent  $a$ . For example, the speed of light in empty space (usually called  $c$ ) is  $c = 299,792,458m/sec$ . This number has 9 significant figures. With 1 significant figure,  $c = 3 \times 10^8m/sec$ . With two significant figures,  $c = 3.0 \times 10^8m/sec$ . The extra zero is very significant: it means that this number is accurate to the next figure. In other words, with one significant figure you don't know if the value is  $3.1, 3.2, 2.8, \times 10^8m/sec$ . Putting that zero in rules out these values. How many significant figures do you need to include to get away from 3? To three significant figures,  $c = 3.00 \times 10^8m/sec$ . Finally, at four significant figures,  $c = 2.998 \times 10^8m/sec$ .

## Dimensional Analysis

There's an even easier way of checking any answer you derive. This is to make sure that the **dimensions** are correct. Dimensions are like units, but a little more general. Dimensions describe the underlying nature of some quantity. For example, "length" is a dimension. Whether you measure a length with the units meter, centimeter, foot, mile, furlong, fathom, it is still a

length. This sounds sort of obvious (and it is), but when you're doing the math, it's easy to get bogged down in details, and forget this simple fact.

In fact, in physics you can often almost guess a formula by just getting the units right. Here's a simple one: Say a car is traveling at some speed  $s$  for a time  $t$ . What is the distance  $d$  it travels? Well, a typical unit for speed is say miles per hour. Thus speed has dimensions of Length/Time. Time has dimensions, well, time. A distance has dimensions length. The only way to get a length out of a speed and a time is to multiply them:

$$d = st.$$

That one is probably too easy. Here's one not so obvious. A car starts at rest and constantly accelerates to a speed  $s_2$  in some time  $t_2$ . What is the distance  $d_2$  it has traveled in the time  $t_2$ ? Even though the exact computation requires a little bit of work (we'll do it in the next lecture), we can get the right answer up to a numerical constant by using dimensional analysis. Just like before, the only way to get a distance from a speed and a time is by multiplying them. Thus

$$d_2 \propto s_2 t_2.$$

The symbol  $\propto$  means "proportional to". It means that there is some unknown number  $k$  on the right-hand side of the equation, i.e.

$$d_2 = k s_2 t_2.$$

By dimensional analysis, we know that  $k$  is just a number: it has no dimensions. Even if we don't know  $k$  (for this problem it turns out that  $k = 1/2$ ), this equation is very useful. It tells us for example that if we double the time, we double the distance, a fact which may be obvious for a car traveling at constant speed, but not so obvious for one accelerating.