

Lecture 2

- Displacement
- Speed
- Average velocity
- Instantaneous velocity
- Gravity and acceleration

Cutnell+Johnson: chapter 2

Most physics classes start by studying the laws describing how things move around. This study goes by the name of **Mechanics**. The reason people usually start here is that most of the notions come straight out of our everyday life: velocity, acceleration, rotation, gravity. The rough idea is simple: forces act on bodies, and the bodies move, rotate, stretch, crack, whatever. Mechanics means how bodies react to forces.

As architects, you'll be particularly interested in a subfield of mechanics called **Statics**. This means studying forces where the body (e.g. a building) **doesn't** move. However, this is often a more difficult problem: all the forces have to balance to prevent the motion. Thus it is first easiest to study motion, without worrying about the forces which cause the motion. Then we will study forces.

Displacement

To really isolate what's going on without additional complications, we'll first study only motion in one dimension. The coordinate describing this dimension we'll call x . The value of x is called the *position*. As you'll see, it doesn't matter at which point you define $x = 0$. This is an arbitrary definition, and changing this won't change any physical results at all.

When an object moves, the most obvious thing to look at is its **displacement**. In words, this means "how its final position has changed from its original position". To express this in

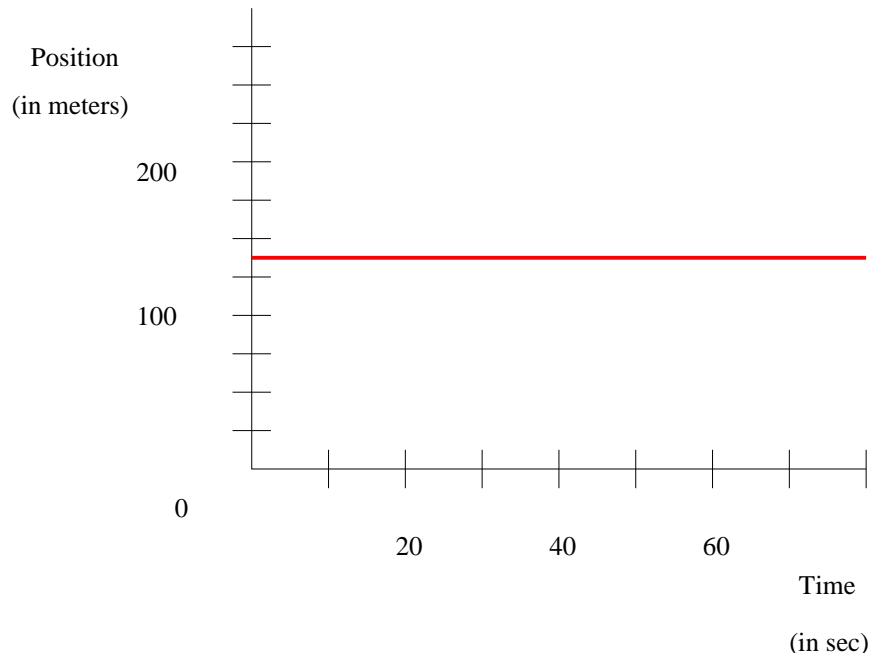


Figure 1: A stationary object

a formula, label the initial position of the object as x_0 , and the final position as x_f . Then the displacement of the object is

$$\Delta x = x_f - x_0.$$

By convention, we define the difference in any variable to be its final value minus its initial value. Note that by this definition, Δx can be either positive or negative. This is a useful piece of information: it tells you whether the object ends to the left or the right of where it was originally. The sign indicates the *direction*; with these conventions, if I move to the left, Δx is negative, if I move to the right, Δx is positive. Remember, we are in one dimension, so the only notion of direction we have so far is left or right. In dimensions greater than one, displacement is a vector. We'll return to vectors in detail later. All you need to know now is that vectors have a *magnitude* and a *direction*. Thus displacement carries more information than distance – distance is always a positive number (in mathematical language, distance is the magnitude of the displacement). The Greek letter capital Delta Δ is often used in physics to denote a difference. It is used in many different contexts, not just in displacement. For example, this class begins at some time t_0 , and ends at some time t_f . The time elapsed in the class is $\Delta t = 75$ minutes.

When studying these problems, a useful tool is to look at a graph. let's look at a graph of displacement vs. time. There's a trivial one in Fig. 1. The displacement doesn't change with time: the velocity is zero.

Speed

The *average speed* for a traveling object is quite simply

$$\text{Average Speed} = \frac{\text{Distance Traveled}}{\text{Elapsed Time}}$$

Problem I leave Charlottesville at 10:00 a.m.(EDT) tomorrow morning, and arrive in Chicago (770 miles away) at 6:00 p.m. (EDT) the following day. What is my average speed?

$$\frac{770 \text{ miles}}{32 \text{ hours}} = 24 \text{ mph}$$

Note I have kept only two significant figures in the answer, because the number 770 miles was given to only two significant figures

An average speed does not say anything about whether I drove quickly, then stopped for a while, or if i started out and drove in for 32 hours without stopping. The numbers in the above formula are just the total distance traveled and the total elapsed time. Thus in this problem, this average is not a particularly interesting one, because it doesn't account for how long I stopped and slept. Thus here's a slightly more interesting problem.

Problem I leave Charlottesville at 10:00 a.m.(EDT) tomorrow morning and drive until 8:00 p.m., covering 500 miles. The next day I leave again at 10:00 a.m. and arrive in Chicago at 6:00 p.m. (EDT) the following day. What is my average speed the first day? the second day? the average while driving both days?

$$\text{first day} \quad \frac{500 \text{ miles}}{10 \text{ hours}} = 50\text{mph}$$

$$\text{second day} \quad \frac{270 \text{ miles}}{8 \text{ hours}} = 34\text{mph}$$

$$\text{while driving} \quad \frac{770 \text{ miles}}{10+8 \text{ hours}} = 43\text{mph}$$

$$\text{for whole trip} \quad \frac{770 \text{ miles}}{32 \text{ hours}} = 24\text{mph}$$

Problem My average speed in climbing the stairs up the Sears Tower (height 1450 feet) is .4 mph. The trip takes 1.5 hours. What is my displacement?

Answer I have traveled

$$\text{distance} = (\text{average speed})(\text{time}) = .4 \text{ mph}(1.5 \text{ hours}) = .6 \text{ miles} = 3200 \text{ feet}$$

However, this is not the displacement, because of course I don't go straight to the top, as if I had taken the elevator. Walking up the stairs requires walking a greater distance, but my displacement in the end is the same. My displacement is 1450 feet, the distance from the top to the bottom.

Average velocity

I've just introduced the quantities displacement Δx and elapsed time Δt . This is pretty simple I hope, so let's move on to something related, but a little subtler. This is to understand **average velocity**. Above I made the distinction between displacement and distance. Distance is the same in any dimension: it's the length of a straight line between two points. We defined displacement above: in one dimension it's $\Delta x = x_f - x_0$, and can be positive or negative. The speed is defined in terms of distance, not displacement. The velocity is defined in terms of displacement:

$$\begin{aligned}\text{Average Velocity} &= \frac{\text{Displacement}}{\text{Elapsed Time}} \\ \vec{v} &= \frac{\Delta \vec{x}}{\Delta t}\end{aligned}$$

I've written the velocity and displacement as vectors to get you in the habit of seeing them this way: in dimensions above one this very important (the book writes vectors by boldface, I denote them with an arrow because it's hard to use boldface on the board). Like displacement, in one dimension a vector is a single number (possibly negative). The velocity lets you do two things with one number: it gives you the speed (which is always positive) and the direction. In mathematical language, the speed is the magnitude of the velocity. The direction is either left or right (or up or down, or whatever). If you're moved to the left (or down), average velocity is negative, moved to the right (or up) it's positive.

A constant velocity corresponds to a straight line on a graph. The steeper the slope, the greater distance the object is traveling in a given amount of time, and hence the greater the speed. Let's now look at an example where the velocity is negative. Being a shy, retiring type, I'm nervous about speaking to a room full of architects while preparing this lecture. So I pace back and forth from one end of the room to another. The graph would look like Figure 2. A plot of the *velocity* versus time is given in Figure 3. Always pay attention to how the graph is labelled.

By looking at these graphs, it should be easy for you to answer the questions: What's the average velocity for the whole thing? What's the average velocity at the beginning until the first time I turn around? What's the average velocity from the first time until the second time? How would it change if I got excited on the last leg and ran back to the computer?

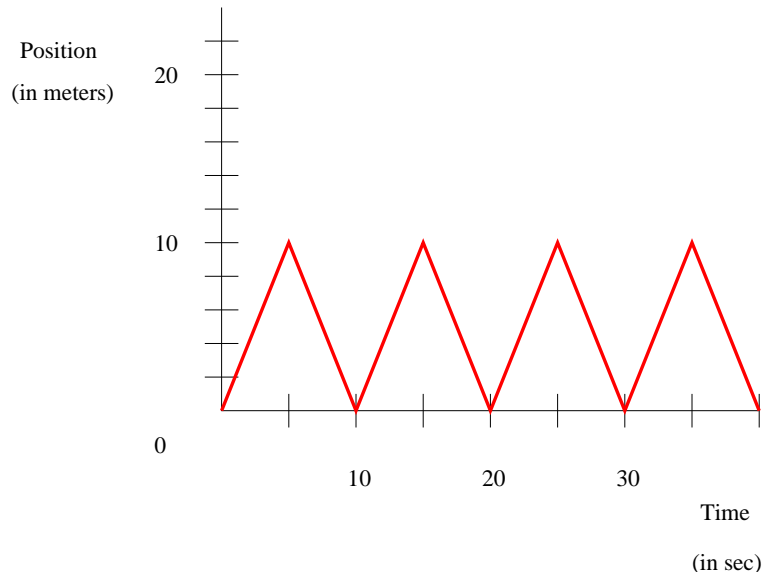


Figure 2: I'm pacing back and forth.

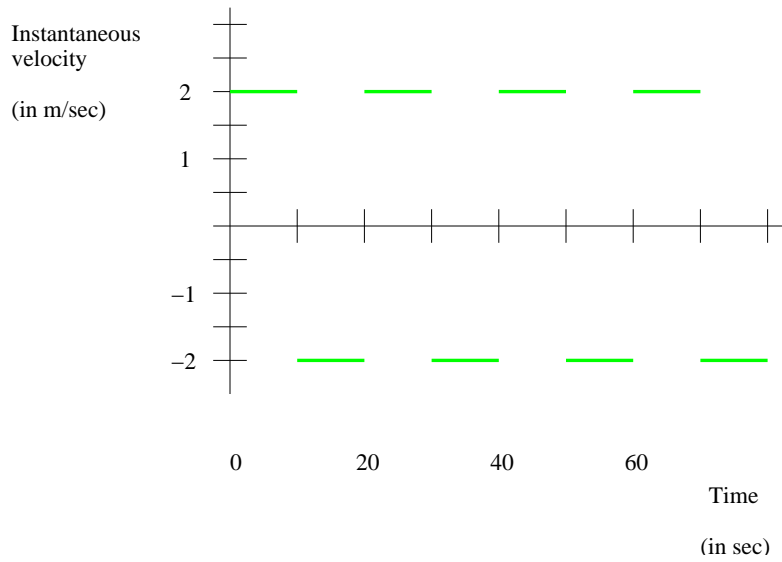


Figure 3: My pacing velocity

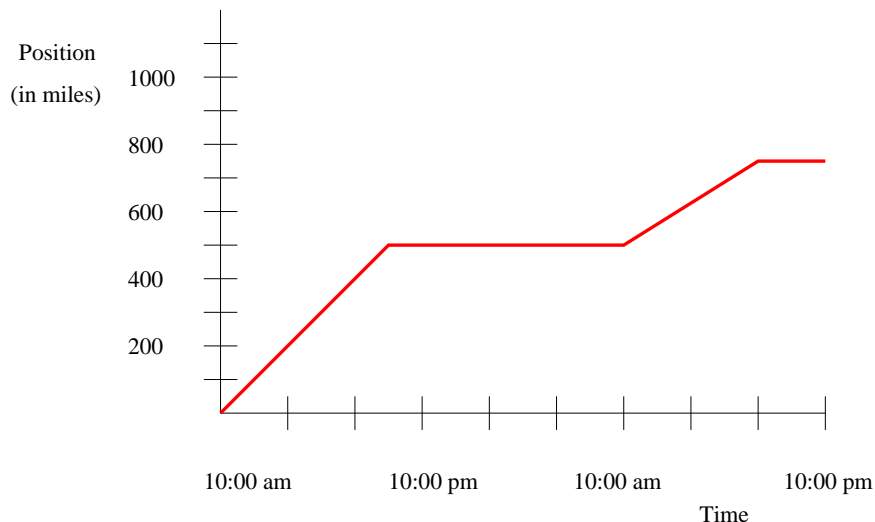


Figure 4: My trip to Chicago

Instantaneous velocity

So far we've just computed average velocity. To compute average velocity, all you need to know is where the object is at the beginning (x_i), and where it is at the end (x_f). Obviously, you need know nothing about what is happening in the middle: whether it stops, backs up, or just moves at a constant velocity. The graphs I drew above, however, give much more information. For example, the graphs tell you what the **instantaneous velocity** is. The instantaneous velocity is the “velocity at a particular point in time”. Likewise, the instantaneous speed is the speed at a particular point in time. In other words, it's what your speedometer reads at any given point in time.

The instantaneous velocity is easy to define precisely. It is merely the average velocity where the elapsed time is very short.

$$\text{instantaneous velocity} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}.$$

The expression \lim is the mathematical term for limit. Even though Δt is getting very small, Δx is also getting small, so the expression remains finite for arbitrarily small Δt . For those of you not scared of calculus, this is called a derivative. That's really all a derivative is: the instantaneous velocity is the derivative of displacement with respect to time.

In the graphs the instantaneous velocity is easy to see. It is precisely the slope at a specific point in time. Thus, for example, for the situation in Figure 2, we can plot the instantaneous velocity versus time. This is displayed in Figure 3.

To see the distinction between average and instantaneous velocity, look at a graph for my Charlottesville to Chicago trip, making the artificial assumptions that I travel in a straight line and that when I'm moving, I move at constant velocity. The graph is in Figure 4. The average velocity on each of the segments would be the same even if the segments weren't a straight line. The instantaneous velocity is the slope at any given point in time.

Gravity and acceleration

When a velocity changes, that's called *acceleration*. The most familiar kind of acceleration comes from gravity: the farther an object falls, the faster it goes.

The definition of average acceleration is very much like that of average velocity, but instead of studying the change in displacement, now we study the change in velocity:

$$\begin{aligned}\text{Average Acceleration} &= \frac{\text{Change in instantaneous velocity}}{\text{Elapsed Time}} \\ a &= \frac{\Delta v}{\Delta t}\end{aligned}$$

Instantaneous acceleration is defined by letting the elapsed time be very short:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}.$$

In most cases discussed in this class, we'll discuss constant acceleration, which means that the instantaneous acceleration is the same as the average acceleration. Say an object has some velocity v_0 at some time t_0 , and is subject to a constant acceleration a . We can use the definition of acceleration above to find an expression for the velocity v at any time t : First write

$$a = \frac{v - v_0}{t - t_0}$$

Then solve for v :

$$v = a(t - t_0) + v_0$$

We can make this equation look a little nicer by defining $t_0 = 0$ (i.e. when you start your stopwatch), so

$$v = at + v_0. \tag{1}$$

This formula I hope is simple: an object starts at time $t = 0$ with a velocity v_0 , and as time goes on, the acceleration means that the velocity changes (if a is negative, the velocity gets lower). It is important to remember that this formula applies only for constant acceleration. A typical

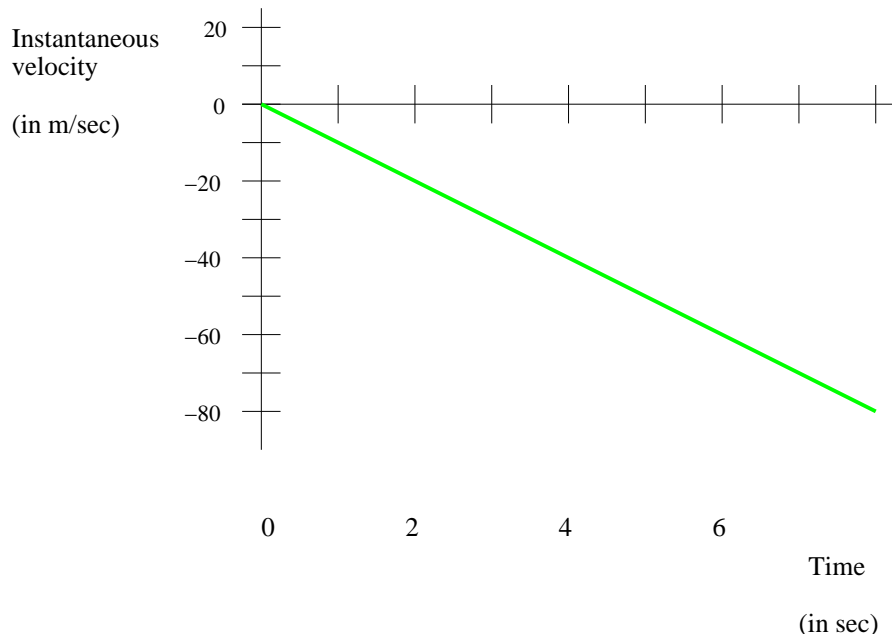


Figure 5: Constant negative acceleration

plot is in figure 5. Just like a constant velocity means that the displacement plot is a straight line, a constant acceleration means that the velocity plot is a straight line.

So now we can ask some simple questions about gravity. First of all, does it act on all objects with the same acceleration? Second, does the acceleration depend on time or velocity, or is it constant?

The first question is easy to answer experimentally. If we drop a piece of paper and a coin, it's obvious which one is accelerated more. However, it's also obvious that air resistance affects the paper much more than the coin. To isolate the effects of gravity from those of air resistance, we put both objects in a vacuum (no air). Then clearly they fall with the same velocity and the same acceleration.

This was noted by Galileo, and allegedly experimentally proven. Galileo purportedly dropped two balls from the Leaning Tower of Pisa, one hollow, one not, but otherwise the same. Thus (if he did the experiment) he showed that the acceleration caused by gravity does *not* depend on mass. You can do analogous experiments to see that on Earth the acceleration caused by gravity doesn't depend on other properties of an object (its composition, its size, its color, whatever). In other words, you a bunch of objects at the same time, and at any time they all have the same velocity (in a vacuum, of course). The conclusion was that in free fall, all objects fall with the same acceleration g .

I say Galileo allegedly did this, because some historians have claimed he never did the experiment. The reason is because he knew what the answer was going to be anyway. Namely, drop two objects, one say of 10 pounds, and one of 20 pounds. People in the old days thought that the 20 pound one would fall twice as fast. Galileo then wondered: what happens if you tie the two objects together? Does the 10 pound ball slow down the 20 pound ball? Does the 20 pound ball pull the 10 pound ball? Does the 20 pound ball pull the 10 pound ball up to its speed? Is it in the middle? Does it depend on the length of the rope tying the two balls together? Or, since the whole thing is now 30 pounds, shouldn't it fall as fast as a 30 pound ball, faster than either individually? All these alternatives sound kind of silly, so Galileo concluded that the effect of gravity was to make all objects fall with the same acceleration g . The nifty vacuum pumps we have make the experiment easy.

Problem How fast will a penny dropped off the top of the Sears Tower be going when it hits the ground? Neglect friction.

Answer There's a formula you can extract from your book which gives the answer directly, but we can already derive it. We know its acceleration is always g . It takes some t to fall, although we don't know t right away. By the definition of acceleration, we know that right before it hits the ground, it is going a speed

$$v_{final} = gt$$

This is the *final* velocity, not the average velocity. By looking at the graph (Figure 5), it's obvious that the average velocity is half the final velocity. In an equation,

$$v_{average} = \frac{v_{final} - v_0}{2}$$

Here $v_0 = 0$ because the penny starts out at rest. Now that we know the average speed, we can find out the distance traveled, from the formula at the beginning of this lecture:

$$d = v_{average}t = \frac{v_{final}}{2}t = \frac{1}{2}gt^2$$

We know d , we know g , this gives us t . But I didn't ask for t , I asked for v_{final} . But looking at the first equation here, we see how the final velocity is related to g . In equation form, we have

$$t = \sqrt{\frac{2d}{g}}$$

$$v_{final} = gt = g\sqrt{\frac{2d}{g}} = \sqrt{2dg}$$

Finally, we can plug in numbers. The distance $d = 1450 \text{ ft}(1 \text{ m})/(3.28 \text{ feet}) = 442 \text{ m}$, so

$$v_{final} = \sqrt{2(442 \text{ m})(9.8 \text{ m/s}^2)} = \sqrt{8664 \text{ m}^2/\text{s}^2} = 93 \text{ m/s}$$

This is about 200 mph. Note that it was easier to wait until the end to plug in the numbers: this is why algebra is a good thing!

Next time we'll do more examples of these kinds of problems.