Lecture 22

- Equilibrium
- Phase Changes
- Heat Transfer

Cutnell+Johnson: 12.6-12.8, 13.1-13.4

Equilibrium

Last time, we studied heat, and how heat can cause a material to either change in temperature, or change in phase. To review, let's first do a problem in temperature change.

Problem A $5\,kg$ chunk of iron at $95.0^o\,C$ is placed in 5.0 liters of water, initially at $20.0^o\,C$. The water and iron reach the same temperature (they *equilibriate*). What is this temperature, given that the heat capacity of iron is $452 J/kg\,^oC$?

Answer Because of the conservation of energy, the heat lost by the iron is gained by the water. Thus the iron goes down in temperature and the water goes down in temperature until the temperature is the same for both. The heat lost by the iron is

$$Q_{iron} = (452 J/kg\,^oC)(5\,kg)(T_{final} - 95^0C)$$

A liter is $10^{-3}m^3$, so the mass of the water is

$$m_{water} = (1000\,kg/m^3)(.005m^3) = 5kg$$

and the heat gained by the water is

$$Q_{water} = (4186 J/kg\,^oC)(20\,kg)(T_{final} - 20^oC)$$

Note that T_{final} is the same for both. To find T_{final}, we use the fact that $Q_{water} = -Q_{iron}$. The reason for the minus sign is that the water is gaining heat, while the iron is losing heat.

$$(4186 J/kg\,^oC)(5\,kg)(T_{final} - 20^oC) = -(452 J/kg\,^oC)(5\,kg)(T_{final} - 95^0C)$$

$$9.26(T_{final} - 20) = 95 - T_{final}$$

$$10.26T_{final} = 95 + 20(9.26)$$

$$T_{final} = 27.3^o\,C$$

So notice the final temperature is much closer to the original temperature of the water than of the iron. The reason is that even though they are both of the same mass, the specific heat capacity of water is much larger.

Phase Changes

If you add heat to a material, the temperature doesn't necessarily have to change. What can also happen is that the phase can change. It takes heat for water to boil, or to melt ice. Similarly, to freeze water into ice requires removing heat. The generic words we use are

evaporation liquid → gas

condensation gas → solid, or gas → liquid

melting solid → liquid

freezing liquid → solid

subliming solid → gas

The formula for the heat required to change phase is simple. It is

$$Q = Lm$$

L is called the *latent heat*. It depends on the material, and which sort of transition is happening. Latent heat is why we sweat: it takes heat from your body to make the sweat evaporate. Having your body lose heat As always, there's a table in your book. One thing to note is that the same latent heat applies for a phase change and its reverse. In other words, it requires heat to melt ice. Then you must extract that same amount of heat to freeze the ice back again.

Problem How much energy does it take to turn $3.0\,kg$ of ice at 0^oC into water at 20^o C?

Answer Initially, when you put in heat, the temperature doesn't change from O^oC. The heat goes into melting the ice. The heat required to melt the ice is

$$Q_{melt} = Lm = (33.5 \times 10^4 J/kg)(3.0\,kg) = 1.0 \times 10^6 J$$

Now the ice is water. The heat to raise the temperature of the water is

$$Q_{raise} = cm\Delta T = (4.19 \times 10^3 J)(3.0\,kg)(20^oC) = 2.5 \times 10^5 J$$

Thus the total heat necessary is $1.25 \times 10^6 J$.

The reason changing phase has an energy change without a temperature change is that the molecules are rearranging themselves. This means that the densiies of a solid and a liquid made of the same material are different. Most of the time, the solid is more dense. However, water is different. Ice above -4^oC is in fact less dense than water. This is why ice floats. This fact is crucial to life, because it means that ponds or lakes freeze from the top down. Then the top insulates the bottom from the cold to some degree, making it more difficult for the entire thing to freeze all the way through. This allows fish to live through the winter.

The word "equilibrium" means more than just that objects are at the same temperature. Equilibrium is a little subtler if the two different materials are different phases of the same material. In other words, can water and ice be in equilibrium? Can water and water vapor be in equilibrium? The answer is yes. The idea is simple. Let's first consider water and water vapor together. This is in fact our daily situation: we have lakes, and there is always some water vapor in the air (unless you're in Death Valley). The two are in equilibrium if the rate at which water is evaporating is the same as the rate the water vapor is condensing.

This idea of equilibrium lets us define what humidity is. 100% humidity means that water and water vapor are in equilibrium. Higher than 100% humidity means that more water vapor is condensing than water evaporating. Lower than 100% humidity means that more water is evaporating than water condensing. A precise number (for example 83% relative humidity) means the fraction of water vapor as compared to its equilibrium value. The equilibrium value depends on the temperature and pressure. This is why it's convenient to give the humidity as a fraction instead of an overall number.

Heat Transfer

We've talked about the effects of heat on materials. Now we'll talk about *how* heat flows. So this in fact goes back to the old way of thinking, where heat was treated as a fluid. It's not a fluid, but it's a reasonable way of thinking about heat flow. There are three kinds of heat flow we'll discuss, called convection, radiation and conduction.

Convection means that the heat is carried from place to place by the material itself moving. This is very prominent in your house: the heat transer occurs from hot or cold air coming in through windows, doors, or seeping through the walls.

When electromagnetic energy to heat, it's called *radiation*. Light, radio, and microwaves are all forms of electromagnetic waves. When these waves hit a material, their energy can be converted to heat. One example you know well is a microwave oven. Another is the heat from the sun. The other way around is also true. Any material at non-zero temperature emits radiation. You've of course seen this. When a metal gets very hot, it starts to emit light. We say it glows red hot, or white hot. Your body is also emitting heat as well. The reason, you're not visible in the dark is that the light you're emitting is out of the visible range. However, there is some light in the infrared, which is how those fancy night sensors work. The amount of heat radiated is proportional to the time, as before. It is also proportional to the surface area (not the volume of the emitter). You would also expect it to depend on the temperature – the higher the temperature, the more heat a body radiates. It turns out that the heat is proportional to T^4, where T is measured in Kelvin. One interesting thing is that in fact there is radiation leftover from the big bang all around us. This is radiation like that coming from a body at $3\,K$. It's not much (it wasn't measured til the '60s), but it's extremely interesting. The reason is that this radiation comes straight from the big bang. Measuring its form is basically doing an experiment on the origins of the universe.

Heat transferred by direct contact is called *conduction*. This means the bulk of the material is not moving. Of course, the molecules inside the material are still moving randomly, but there is no net flow of these molecules. In other words, the air is still, the water isn't flowing. Yet heat can still be transferred. This occurs because the molecules in one material collide with the molecules in the other material. In such a collision, kinetic energy can be transferred from one molecule to another, as we saw when we did the chapter on momentum.

Materials which conduct heat well are called thermal conductors, while those which conduct heat poorly are called thermal insulators. Most (but not all) metals are good thermal conductors. This is why metal is usually cold to the touch, even though the metal is the same temperature is the room air. Your body temperature is warmer than room temperature, so heat flows to whatever you're touching. A good thermal conductor takes the heat out of your body quicker, and thus cools your skin off quicker. Air is a very poor thermal conductor, which is why styrofoam and goose down are thermal insulators. Another famous thermal insulator is asbestos, which (unfortunately) was used in millions of buildings.

A simple formula describes heat conduction. Let's consider a bar of length L and cross-sectional area A. One end held at a temperature T_1, and the other held at a temperature T_2. Heat will flow from the hotter end to the colder end. First of all, you expect that the longer the time, the more heat flows. Thus $Q \propto t$. Second, you expect that that the greater the temperature difference, the quicker the heat will flow. $Q \propto \Delta T$. Third, the wider the bar is, the more heat will flow. Thus $Q \propto A$. Finally, the longer the bar is, the longer it will take heat to flow. Thus $Q \propto 1/L$. Putting this all together gives

$$\frac{Q}{t} = \frac{A\Delta T}{\mathcal{R}}$$

The constant $\mathcal{R}$ is called the "R-value" of the material; it is proportional to length (your book writes $\mathcal{R} = L/k$, where k is called the thermal conductivity). The right-hand-side describes the amount of heat transported per time. Thus the larger $\mathcal{R}$ is, the less heat is transported, and so $\mathcal{R}$ is a measure of how insulating the material is.