

## Lecture 23

- Charge and the Coulomb Force
- Electric Potential
- Current

Cutnell+Johnson: 19.1-19.2, 20.1-20.2

### Charge and the Coulomb Force

Describing the electric force is quite simple. We saw in gravity that the strength of the gravitational force is proportional to the mass. The strength of the electric force on a body is proportional to a quantity called the *electric charge*. Two bodies with charges  $q_1$  and  $q_2$  have a force between them. The law describing this force is called *Coulomb's Law*, and is

$$F = k \frac{q_1 q_2}{r^2}$$

The MKS unit of charge is the Coulomb, which is usually abbreviated by  $C$ . The constant  $k$  plays the same role as  $G$ . Here,  $k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ . One big difference between the electric force and the gravitational force is that the electric force can be either attractive or repulsive, while gravity is always attractive (there is no anti-gravity except in science fiction). This is because charge can be either positive or negative. As opposed to gravity, charges of the same sign *repel*. Thus if both  $q_1 > 0$  and  $q_2 > 0$ , or if both  $q_1 < 0$  and  $q_2 < 0$ , the force is repulsive. On the other hand, if  $q_1 > 0$  and  $q_2 < 0$ , then the force between these charges is attractive. In all cases, the force is given by Coulomb's Law.

The fundamental particles which carry charge are called the proton and the electron. The proton has charge  $+1.60 \times 10^{19}C$ , while the electron has exactly the opposite charge:  $-1.60 \times 10^{19}C$ . These charges are commonly called  $+e$  for the proton,  $-e$  for the electron. Note that in one Coulomb of charge, there are  $6.25 \times 10^{18}$ . Every atom normally has the same number of electrons and protons, so an atom is said to be electrically neutral, meaning that it has charge zero. However, there are various ways of knocking one or more electrons away from an atom. Say one electron is knocked off. Then the atom has positive charge  $1.60 \times 10^{19}C$ . This charged atom is called an *ion*.

Electricity is usually carried by electrons freed from atoms. Something which allows electrons to travel without much obstruction are called *electrical conductors*, while those which do not flow well are called *electrical insulators*. Many (but certainly not all) metals are conductors, while many. The reason electrons flow and not the ions is simple: the electron is much much lighter and thus is easier to move (a proton has more than 1000 times the mass of an electron). It is also interesting to note that the electron actually was named after electricity, not vice versa. This accounts for why an electron has negative charge – if they had known that it was the electrons flowing, then presumably they would have defined charge so that electrons would have positive charge and protons negative.

The electric force is very much like gravity. Both forces act at a distance. Charge is like mass, although mass is always positive and the force of gravity is always attractive. In fact, the general law of gravitation is very much like the Coulomb's Law. The force between two bodies of mass  $m_1$  and mass  $m_2$  a distance  $r$  apart is

$$F = G \frac{m_1 m_2}{r^2}$$

where  $G = 6.67 \times 10^{-11} Nm^2/kg^2$ , The universal gravitational constant  $G$  plays the same role as  $k$  in Coulomb's Law. Like  $k$ ,  $G$  is the same anywhere in the universe. This is unlike little  $g$ , which depends on where you are. On the surface of the earth, the gravitational force is between a body of mass  $m$  and the earth, of mass  $m_e$ . The distance is the distance from the center of the earth, which is the radius of the earth  $r_e$ . Thus on the surface of the earth, you can simplify the gravitational law to

$$F = G \frac{mm_e}{r_e^2} = m \left( \frac{Gm_e}{r_e^2} \right) = mg$$

This defines  $g = Gm_e/r_e^2$ . Plugging in the appropriate values, we recover our old friend  $g = 9.80 m/s^2$ . This formula explains why gravity is less on top of a mountain than it is at sea level – you must use a slightly higher value for  $r_e$  in this equation. The effect isn't huge, but it is measurable.

## Electric Potential

We've discussed several kinds of potential energy in this class. There's been elastic potential energy, and gravitational potential energy. Since the electric force is so similar to the gravitational force, it shouldn't be surprising that there is an electric potential energy. All three of these forces are conservative. For example, if you pull two masses apart, the energy to do so is put into potential energy. (Pulling masses apart is what you're doing when you lift something up from the earth, as we did many times). You can get this potential energy back by letting the object fall, converting the potential energy to kinetic energy. Similarly, if you push two charges (with the same sign charge) together, it takes energy. This energy is converted into electric potential energy. When you let the charges go, the charges spring apart, converting the potential energy into kinetic energy.

For the electric force, it is common to use *potential* instead of potential energy. The potential is simply related to potential energy. If a charge  $q_0$  has electric potential energy  $PE_{elec}$ , then the potential  $V$  is defined by

$$V = \frac{PE_{elec}}{q_0}$$

The MKS units for electric potential is the volt, abbreviated by  $V$ . The relation between volts, Joules and coulombs is  $1V = 1J/C$ . In the context of electricity, it is very common to refer to an electric potential as the *voltage*. The main reason for discussing the potential instead of the potential energy is that it is independent of the properties of the object. In other words, it is independent of the charge  $q_0$ . This is why we discuss the potential of a battery, not its potential energy: the potential energy depends on how many electrons are moving around in a circuit, while the potential does not.

Note that one could define a gravitational potential as well as  $PE_{grav}/m$ . Thus on the surface of the earth, the gravitational potential would be  $gh$ .

Also note that for both the electric and gravitational potential energies, what is important is the change in potential energy, as the charge or mass moves from one point to another. All physics depends only on  $\Delta PE$ . For this reason, electrical potential  $V$  is often called the potential *difference*. A  $9V$  battery means that the potential difference between the two terminals is 9 volts.

**Problem** In a television picture tube, electrons strike the screen after being accelerated from rest through a potential difference of  $25000V$ . What speed do they have right before they hit the screen?

**Answer** Just like in the gravitational problems, overall energy is conserved. Thus

$$\Delta KE + \Delta PE = 0$$

Consider a single electron, of charge  $-e = -1.6 \times 10^{-19}C$ , and mass  $9.1 \times 10^{-31}kg$ . Then  $\Delta PE = -eV$ , while  $\Delta KE = mv^2/2$ . Therefore

$$\frac{1}{2}mv^2 = eV$$

Solving for  $v$  gives

$$v = \sqrt{\frac{2eV}{m}} = 9.4 \times 10^7 m/s$$

Note that this is very close to the speed of light ( $3.0 \times 10^8 m/s$ ). It turns out that near the speed of light, the formula  $1/2mv^2$  for kinetic energy isn't right, so this answer in reality is slightly modified.

## Current

In this last example, we've seen that if you put charges in a voltage, they move. This is nothing different from when you put a mass in gravity, it moves. However, we give a special name to charges in motion. It is called *current*. The current is defined as

$$I = \frac{\Delta q}{\Delta t}$$

The current at any point is the amount of charge  $\Delta q$  which passes by that point in a time  $\Delta t$ . The MKS unit is called the ampere, or amp for short. It is abbreviated by  $A$ , and is

$$1A = 1C/s$$

There are two common kinds of current: *direct current* (DC), and *alternating current* (AC). With direct current, the charges all move in the same direction, while with alternating current they move back and forth. Usually, AC current is given by a sine wave:  $I_{AC} = I_0 \sin(\omega t)$  This is why we say the wall current is 60 Hz: the current goes through 60 cycles in a second (recall that  $\omega = 2\pi f$ ).

Current is almost always carried by electrons. Because the electron has a negative charge, this means that with the above definition of current, if you define  $I$  to go in one direction, the electrons are moving in the other.