Lecture 24

- Resistance
- Power
- Circuits

Cutnell+Johnson: 20.2-20.4

Resistance

As we saw in the example of a picture tube, the higher the voltage, the faster the charges go. And the faster the charges go, the higher the current. Thus you might expect that $V \propto I$. This is called *Ohm's Law*. It reads

$$V = IR$$

The constant of proportionality is called the *resistance*. Its MKS units are the ohm, which is usually abbreviated by the Greek letter capital Omega Ω. From Ohm's Law, it follows that

$$1\Omega = 1V/A$$

Like the bending of metal, the linear formula is only an approximation. However, it is valid in most situations.

The nice thing about resistance and Ohm's Law is that the resistance of a given object is more or less independent of the current going through it. This is why you can just go to a hardware store and buy resistors, which have a given resistance (if you've ever seen them, they often have these colored stripes painted on them, which codes their resistance). The resistance of an object depends of course on which material it's made of, but it depends on the shape as well. Say current flows from one end to another. The longer the object, the greater the resistance: the current has to make its way through more material. The bigger the cross-sectional area A, the less the resistance: the current has more ways to make it through, making it easier. This results in the formula

$$R = \rho \frac{L}{A}$$

ρ is called the *resistivity*. The resistivity depends only on the kind of material, and not the shape. Thus resistance and resistivity are independent of I and V, while ρ is also independent of the shape of the wire as well. So really when we say a material is a good conductor, we mean it has a low resistivity. A long enough copper wire can have a substantial resistance.

Circuits

Now that we know about voltages, currents and resistance, we can talk about *circuits*. A circuit is simply a closed loop of current. There is a battery to supply the voltage, a wire to carry the current, and something attached to the wire (like a washing machine, a light bulb, etc.) A material with resistance is called a *resistor*. Insulators have a large resistance, conductors a small one. You put resistors in a circuit to lower the current in that part of the device. The resistance of copper is very small relative to most things, so it is common to just neglect the resistance in the wire, and just worry about the other things (like the light bulb or washing machine) connected to the circuit. The conventional way to draw a circuit is to draw the $+$ and $-$ terminals of the battery, a line for the wires, and a squiggly line for any resistors on the circuit. Circuits may also have switches, which allow you to stop the current by breaking the wire. You may notice that sometimes light switches spark. That happens when the switch is not quite on, but close enough so that the current can jump across the air. Another common part of a real circuit is a fuse. The point of a fuse is that it can carry only so much current. It blows up if you put too much current in it, and breaks the circuit. Back in the old days, people used fuses in their houses, but circuit breakers are more convenient, because they can be easily reset. Still, fuses are used in most electronic devices – it's better to blow an fuse than to blow up the whole thing.

Problem A wall outlet has a fixed *voltage*, which is $120\,V$ in the USA. The current varies, depending on the resistance of the objects attached. At what resistance will a $2A$ fuse blow?

Answer

$$R = \frac{V}{I} = \frac{120\,V}{2\,A} = 60\Omega$$

For any resistances lower than 60Ω, the fuse will blow. By the way, the wall voltage is significantly higher in Europe, which is why people more frequently get electrocuted. You have a fixed resistance, more or less, so if you stick your finger in a wall socket at twice the voltage, twice the current flows through you (bad).

When drawing a circuit, it is important to keep track of which voltages and which currents you're talking about. The current must be the same in a single wire: electrons are not accumulating anywhere, so they must be moving by at the same rate. If a wire splits into two, then some of the current goes one way, some goes the other. A voltage, on the other hand, is defined between two points. We say a "voltage drop", or a "potential difference" is present between these two points. What this means that there is a difference in potential energy between these two points. This is why you often heard it called a voltage "drop"; a charged object gains kinetic energy in a voltage drop just like a ball dropped in a magnetic field.

Power

I keep emphasizing how a voltage difference is related to the difference in potential energy. When a wire has resistance, the energy goes to getting the charges through. It's like pushing your way through a crowd. By the time you've made it through the crowd, you've run out of energy, and need another push. Energy overall is conserved: this energy goes to heating the resistor. In a light bulb, some of the energy is converted to light as well, although incandescent bulbs are very inefficient: most of the energy goes to heating.

When there is energy flow, there is power. Power is as always

$$P = \frac{\Delta E}{\Delta t}$$

Here we know that the change in energy between the two terminals is due to a change in voltage. Thus $\Delta E = \Delta q V$. This energy is converted to kinetic energy, but because of resistance, it is converted to other forms as well. Plugging this into the formula for power gives

$$P = \frac{(\Delta q)V}{\Delta t} = \frac{\Delta q}{\Delta t}V = IV$$

The MKS units of power are, as we've seen, the Watt, which is a J/s. Here, we see that one watt is one volt-ampere as well. You can check that this is consistent with the definitions of the units.

Since $V = IR$ as well, we can rewrite the power formula in the forms

$$P = VI = I^2R = \frac{V^2}{R}$$

Problem What is the resistance of a $50\,W$ light bulb?

Answer The $50\,W$ is based on the $120\,V$ in the US. To find the resistance, we use $P = V^2/R$.

$$R = \frac{V^2}{P} = \frac{(120\,V)^2}{50\,W} = 288\,\Omega$$

Resistance at low temperatures

The resistance of an object is not a sacred number. We saw earlier how the resistance of a wire depends on its length and cross-sectional area:

$$R = \rho \frac{L}{A}$$

where ρ is the called the resistivity, and depends on the material. The resistivity can depend on the temperature. As we saw, the atoms are moving slower at a lower temperature. Therefore the obstruct the current less at lower temperature, and the resistance goes down.

There is a remarkable phenomenon which happens at low enough temperature in some materials. Below some temperature, called the critical temperature, the material has no resistance at all. I don't mean a very small resistance, but actually zero. This is called *superconductivity*. Current will persist in a superconductor without a battery for literally years. Oddly enough, superconductivity usually occurs in materials in which are *not* good conductors at normal temperatures. For example, copper does not superconduct, no matter how low the temperature.

There was a great breakthrough in physics about 13 years ago, when superconductors with a relatively high critical temperature (above liquid nitrogen's temperature) were discovered. This is important because liquid nitrogen is easy to make (just keep pumping to lower the pressure: recall the ideal gas law). People envisioned power lines with no resistance. Unfortunately, this hasn't proven practical yet, basically because no one has yet figured out how to make wires of this material which can carry a lot of current.