Lecture 25

- Parallel and Series Resistances
- Alternating Current

Cutnell+Johnson: 20.5-20.8

Parallel and Series Resistances

Last time I pointed out that the resistance of a wire increases with length, but decreases with cross-sectional area. The same ideas hold for circuits with multiple resistors. Say put a resistor of resistance R_1 after one of resistance R_2, so that the current through the first one I_1 is the same as the second I_2. This is known as putting the resistors in *series*. Say we connect these resistors in series to a batter of fixed voltage V. The current is *not* have $I_1 = V/R_1$ and $I_2 = V/R_2$. Ohm's Law applies between any two points, so we have $I_1 = V_1/R_1$, and $I_2 = V_2/R_2$. V_1 is the voltage drop across the first resistor, while V_2 is the voltage drop across the second. Since the resistors are in series, $I_1 = I_2$, so

$$\frac{V_1}{R_1} = \frac{V_2}{R_2}$$

Since voltage is defined as between two points, the total voltage $V = V_1 + V_2$. You can use these two relations to solve for V_1 and V_2 and hence I, giving

$$I_1 = I_2 = \frac{V}{R_1 + R_2}$$

Note that this means you can view the two resistors as being a single resistor with a resistance

$$R_{series} = R_1 + R_2$$

In other words, if I put the series resistors together in a box, the box has the total resistance $R_1 + R_2$.

Putting resistors in series is like making a wire longer and increasing its resistance. To lower resistance, we can put resistors in *parallel*. Two resistors R_1 and R_2 in parallel have the same voltage across each: $V = V_1 = V_2$. This requires splitting the current: the current coming out of the battery I i s split into two. Since overall charge is conserved, $I = I_1 + I_2$.Thus resistors in series have the same current through them, resistors in parallel have the same voltage across them. As always, we have $I_1 = V_1/R_1 = V/R_1$ and $I_2 = V_2/R_2 = V/R_2$. Plugging this in gives

$$I = I_1 + I_2 = \frac{V}{R_2} + \frac{V}{R_2}$$

Thus

$$V = I\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Thus if you were to put the two resistors in parallel in a box, the box has total resistance

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Problem A simple example of a parallel circuit is a three-way light bulb. It consists of two resistors in parallel. In the dimmest position, the current goes through the resistor R_1. In the middle position, the current goes through R_2, and in the brightest position it is going through both resistors. Say we have a $50/100/150\,W$ bulb. What is the total current in the $150\,W$ position?

Answer First, let's figure out the individual resistances. We've already done this for the $50\,W$ setting, because this consists of a single resistor with $120\,V$ across it. This leads to $R_1 = 288\Omega$. The value of R_2 is found from the same $P = V^2/R$ we used before. Since the power is twice as large, the resistance must be twice as small, so $R_2 = 144\Omega$. We can get the curre nt in the $150\,W$ setting in two equivalent ways. Note that because the resistors are in parallel, $V_1 = V_2 = 120\,V$. We have

$$I_{tot} = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{120\,V}{288\Omega} + \frac{120\,V}{144\Omega} = .417\,A + .833\,A = 1.25\,A$$

Equivalently, we can use the formula for parallel resistances.

$$\frac{1}{R_{parallel}} = \frac{1}{288\Omega} + \frac{1}{144\Omega} = .0104/\Omega$$

This gives $R_{parallel} = 96\Omega$. Then

$$I_{tot} = \frac{V}{R_{parallel}} = \frac{120\,V}{96\Omega} = 1.25\,A$$

Problem A heater and a lamp are connected in parallel to the same $120\,V$ outlet. Together they use a total of $84\,W$ of power. The resistance of the heater is 600Ω. What is the resistance of the lamp?

Answer It's very useful to draw the circuit. Because the heater and lamp are connected in parallel across the battery, the voltages across them are the same $V_h = V_l = 120\,V$. Since the resistance of the heater $R_H = 600\Omega$, the current in the heater must be

$$I_h = \frac{V_h}{I_h} = \frac{120\,V}{600\Omega} = .2\,A$$

This means that the heat disspipated in the heater is

$$P_h = V_h I_h = (120\,V)(.2\,A) = 24\,W$$

You could have used $P_h = V_h^2/R_h$ to get the power directly. The power dissipated in the lamp must therefore be $P_l = 84\,W - 24\,W = 60\,W$. Now we can use $P_l = V_l^2/R_l$ to get

$$R_l = V_l^2/P_l = (120\,V)^2/60\,W = 240\Omega$$

The lamp has less resistance, so for a fixed voltage, draws more current and hence draws more power.

Alternating Current

All the formulas above apply for DC current. For AC current, the physics is the same, but the formulas need to be treated a little more carefully. The point is that the voltage and the current are changing with time. In the US, the wall current is

$$V = V_0 \sin \omega t$$

Most of the formulas are the same. However, the formula for power is different. It is changed to

$$P_{average} = \frac{1}{2} V_0 I_0$$

The reason for the 1/2 is that because I_0 and V_0 are the *peak* values of the voltage and current. Thus to get the power dissipated in the circuit, you need to average over all the values. This results in the factor of 1/2 in the power. To compensate for this, people often define the "root-mean-squared" voltage and current:

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad\qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

Then Ohm's law still applies, so $V_{rms} = I_{rms} R$, but now

$$P_{average} = V_{rms} I_{rms}$$

The upshot is that if you use the rms voltage and current, all the old formulas imply. In fact, we've already used this fact. I've said that wall voltage is $120\,V$. This is rms voltage, which is why all the formulas I've used are correct.

Capacitance

Resistors certainly are not the only thing that you can put in a circuit. Another common circuit element is a *capacitor*. This usually consists of two parallel plates, although you can make them quite small. What these parallel plates do is store charge. If there is a potential difference V across the two plates, the amount of charge stored is proportional to V. This gives the formula

$$Q = CV$$

where the constant of proportionality C is known as the capacitance. The higher the capacitance, the more charge stored. Capacitances have many uses. One is to store power in case of a power outage: this is why your VCR can keep running for a little while, even if the power goes out. Another is actually inside a computer, in the RAM: the 1's and 0's the computer is using while running is are stored by charging or discharging a capacitor. For AC circuits, capacitors are very useful. The reason is that they are always charging and discharging, because the voltage is changing. However, because of the time delay, the resulting current is out of phase with the voltage (it has a peak when the voltage is zero, and is zero when the voltage is peaking).