**Lecture 5**

- Newton's First Law
- inertia and mass
- Newton's Second Law ($F = ma$)
- Net force
- Newton's Third Law (action-reaction)

Cutnell+Johnson: chapter 4.1-4.5

In the last lecture and in numerous demonstrations we've seen how the equations we've derived apply to any individual direction. For example, gravity acts to pull objects down. If it is the only force in the problem, gravity affects only vertical motion: gravity leaves the horizontal motion untouched. The dart gun with falling target is an example of this. Both the dart and the target fall vertically with the same acceleration: the horizontal velocity makes no difference.

I keep using the word "force". We've avoided discussing forces in detail so far, just concentrating on the motion itself. We now need to start discussing the actual forces which cause the motion. This will enable us soon to be able to understand how forces appear in problems without any motion: e.g. a building standing still.

Everybody has an intuitive notion of force: you push or pull something, and it moves. It gets a little bit trickier if there's more than once force: if I push on the wall, it pushes back on me, and nothing moves. Yet we still say there are forces involved: there's a difference between pushing on the wall and not doing anything: maybe if I push long enough the wall and the building will fall.

Thus we need to understand how to make the notion of force precise. Because this is pretty important for physics, the basic notions are glorified by calling the **Newton's Three Laws**. All of them are pretty simple, but putting them together allows us to make precise (or to use the word I like, "quantify") what it means to have a force.

Newton's First Law makes precise something we've already seen, the fact that without forces, thing just keep doing what they're doing. The law is

An object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force.

Part of this law is obvious. If something is not moving ("a state of rest"), and you don't do anything to it ("compelled to change by a net force"), it just stays there. The other part is simple, but maybe not as obvious. It says that if an object is moving in a straight line with a constant speed, it will keep doing that unless a net force is applied. This is not completely in accord with our intuition, because we always see things slow down and stop. What the law means is that to slow down and stop a moving object, some sort of force needs to be applied. As we'll learn, friction is a force, whether it be applied by air, or by rubbing along the ground. If there is no friction or some other force, the object will keep moving indefinitely. This is why spaceships don't need much power to operate. As opposed to planes in our atmosphere, they don't have to overcome gravity and air friction.

Related to this law is a fact we've already used. Here's my Law 1A:

Force is a vector.

This means that a force has a magnitude (how strong the force is) and a direction (which way the force is trying to make the object go). Thus in one dimension, force has a sign. If up is defined to be positive, than the force of gravity is always negative. You can think of this sign as being the difference between pushing and pulling. Note that just like the acceleration, the sign of the force does not have to be the same sign as the velocity. When a ball is going up, the force of gravity is still negative, even though its velocity is positive.

In dimensions higher than one, the fact that force is a vector has even bigger consequences. We can study each component of the force, and Newton's Laws apply to each component individually. In fact, we've already used Newton's first law and law 1A, when we studied acceleration in two dimensions. Let's go back to the ball dropped out of the airplane, which I discussed in detail in the notes for Lecture 5. Let's just look at forces and motion in the individual directions. First, let's do the horizontal direction. Since gravity acts only in the vertical direction and we are neglecting air friction, there are no forces in the horizontal direction. Thus in the horizontal direction, Newton's first law is relevant. It means that its motion in the horizontal direction does not change. Since the ball is on the plane to begin with, it is traveling with the plane's velocity. Dropping it doesn't change any forces in the horizontal direction, so its horizontal velocity doesn't change.

An illustration of this is also given by throwing a ball up in a train car. If the train is moving at a constant velocity, and there are no forces like bumps on the track, it is easy to throw a ball up and down on a train. To you on the train, it seems no different from if you were standing on the ground (because it isn't any different). You throw it up, it comes straight down. However, to an observer on the ground, it looks much different. You're moving at some constant velocity, you throw the ball up, the ball and you move at the same horizontal velocity, and then you catch the ball. The trajectory of the ball is an arc. This an example of Newton's first law: the horizontal velocity is unaffected by any force, so you and the ball just keep moving. This looks sort of amazing in the demo, but the demo is really nothing more than this fact.

A word related to Newton's first law is *inertia*. Inertia is this tendency to continue as is in the absence of forces (stay at a constant velocity). Everything seems to have inertia, but we're very familiar with the idea that some things have much more inertia than others. This means it takes more force to change their state: it is a lot harder to get a train moving (or to stop an already-moving one) than it is to get a person moving. To measure the inertia of an object, we use a quantity called mass. Every object has a mass, and in the MKS system the mass is measured in kilograms. The larger the mass, the more force it takes to get it moving. Mass is not a vector, it is just a number (with units of course).

Note that the unit of mass in the English system is **not** pounds. As we'll see soon, pounds are a unit of force, not mass. The English unit of mass has the very apt name of *slugs*. The higher your mass in slugs, the more sluggish you are.

**Newton's Second Law** ($F = ma$)

The point of the first law is that objects in motion continue that motion if left alone. Moreover, the mass of an object is a measure of how much inertia it has. In other words, the mass is a measure of how hard it is to change an object's path. The point of the second law is to make precise how to *not* leave an object alone. In other words, I will define forces.

Newton's second law explains how the *net* force relates to motion. The net force is the sum of all forces. Since force is a vector, this is a vector sum: you need to add each of the components individually. The net force is denoted $\Sigma\vec{F}$: the Greek letter $\Sigma$ is often used to mean "sum". Newton's second law is

> When a net external force $\Sigma\vec{F}$ acts on an object of mass $m$, the resulting acceleration $a$ obeys the equation
>
> $$\Sigma\vec{F} = m\vec{a}$$
>
> or equivalently
>
> $$\vec{a} = \frac{\Sigma\vec{F}}{m}.$$

In other words, the acceleration is in the direction of the net force, and its magnitude is the magnitude of the net force divided by the mass, i.e. $a = \Sigma F/m$. Remember that $\vec{F}$ and $\vec{a}$ are vectors, while $m$ is not.

Several words in the law are very important. The first is *net*. When applying the second law to get the acceleration of an object, you must use the net force. If I'm pushing on opposite sides of an object with forces of the same magnitude but opposite directions, the net force is zero and the object does not move. Situations where the net force vanishes are called *static*. The word "external" is also important. An internal force is one an object exerts on itself. If I pull on my arm, I can make my arm move, but not my whole body (ever try to move on ice without skates or cleats?). We'll come back to this issue when we discuss the center-of-mass next time.

Because of the second law, the MKS unit of force therefore must be formed from the units for acceleration and mass. It is

$$kg\left(\frac{m}{s^2}\right)$$

Since this is a bit unwieldy, it gets its own name, the Newton, abbreviated by $N$, so

$$1\,N = 1\,\frac{kg\,m}{s^2}$$

**Problem** How much force do I need to put on a $8\,kg$ bowling ball to give it an initial $3m/sec$ velocity down the lane within .5 seconds? How much force do I need to give the bowling ball the same velocity, but within .25 seconds?

**Answer**. In the first case, magnitude of the ball's acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{3m/s}{.5s} = 6\,m/s^2$$

so the force is

$$F = ma = 8\,kg(6m/s^2) = 48N.$$

In the second case, the magnitude of the acceleration is

$$a_2 = \frac{3m/s}{.25s} = 12m/s^2,$$

so

$$F_2 = ma_2 = 8\,kg(12m/s^2) = 96N.$$

So notice to accelerate the ball in half the time, I need to apply twice the force. The force depends not just on how fast the ball ends up going, but also on how long I take to get it that way.

Let's do a problem with more than one force in it (but still in one dimension)

**Problem** I'm in a tug of war with my children: I can put $200N$ on their favorite toy, while my daughter pulls on it with $130N$ and my son with $110N$ (in the opposite direction from me, of course). What is my acceleration, if I weigh $75kg$?

**Answer** We'll define the plus direction to be in my direction. Thus the force I apply is $200N$, while the forces my children apply are in the other direction, so they are $-130\,N$ and $-110\,N$. The net force is therefore

$$\Sigma F = 200\,N - 130\,N - 110\,N = -40N$$

The acceleration is therefore

$$a = \frac{\Sigma F}{m} = \frac{-40\,N}{75\,kg} = -.53\,m/s^2$$

The acceleration is the negative direction, which means that my children are winning the battle. Note that if to get the final units $m/s^2$, one needs to remember that a Newton is $kg\,m/(s^2)$.

**Newton's Third Law**

Newton's Third Law is also pretty intuitively obvious, but has many important consequences, particularly for buildings. It is

> Whenever one body exters a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body.

The traditional way of saying this is "For every action there is an equal and opposite reaction." Colloquially, you can say, "if you push on something, it pushes back".

We've already seen one example of this, in the idea of net force. You cannot sail your own boat: even if you put a force on the air and the air puts a force on the sails, the sails put a force on the air, and the air puts a force back on you. The net force for you, the air, and the sails is zero. Thus you can push the sails away from you, but you cannot make the combined system move.

**Simple Problem** An $80\,kg$ hockey player hits a $.5\,kg$ puck, which gets an accleration of $100m/s^2$. What is his resulting acceleration?

**Answer** The puck is hit by a force of $(.5\,kg)(100\,m/s^2) = 50N$, so the player must react with a force of $-50N$ (same magnitude, opposite direction). His acceleration is $a = (-50\,N)/(80\,kg) = .5m/s^2$.

**Trickier Problem** The same hockey player hits the same puck so hard it splits in two equal pieces. The pieces fly off with a 90 degree angle between them, and an acceleration of $150\,m/s^2$ each. What is his resulting acceleration?

**Answer** Let's say that one piece is traveling along the $x$ axis and the other along the $y$ axis. Thus the accleration of the first is $(150m/s^2, 0)$, while that of the second is $(0, 150m/s^2)$. Thus the forces are

$$\vec{F}_1 = (-37.5N, 0) \qquad \vec{F}_2 = (0, -37.5N)$$

The force on the hockey player is the opposite of the sum of the two forces, namely

$$\vec{F}_{player} = (-37.5N, -37.5N)$$

or if you wish, has a manitude $37.5\sqrt{2}N = 53N$ in the direction halfway in between the $-x$ and $-y$ axes.