Lecture 6

- Weight
- Tension
- Normal Force
- Static Friction

Cutnell+Johnson: 4.8-4.12, second half of section 4.7

In this lecture, I'm going to discuss four different kinds of forces: weight, tension, the normal force, and friction. We'll study how these forces cause acceleration, or cancel to give a static problem.

Weight

The weight of an object is *not* the same as its mass. The weight is defined as the force exerted by gravity. Hence far out in space, where there is very little gravity, we say an object is weightless, not massless. On the surface of the earth, the effect of gravity in the absence of other forces is to give an object an acceleration of g towards the center of the earth. Thus the weight W of an object on the surface of the earth is

$$W = mg \tag{1}$$

(we often say "weight" to mean the magnitude, even though weight is a vector – it is understood the direction is towards the center of the earth). The formula for weight $W = mg$ is valid even if the object is not accelerating or moving. Weight is the force of gravity on an object.

The English unit of force (or weight) is the pound. So we see why my identification in Lecture 1 of pounds with kilograms was not correct: they don't have the same dimension! Pounds measure a force, while kilograms measure mass. I have the same $75kg$ mass in outer space, but am weightless.

Problem I (still at $75\,kg$) am testing a new experimental aircraft. While accelerating intensely, someone says that I underwent a "force of 4 g's". What does this mean?

Answer Saying a force of 4 g's really isn't right, because g is an acceleration. What people mean is that the acceleration I'm undergoing is 4 times that of gravity. Thus magnitude of the force acting on me is

$$F = mg = (75\,kg)(4 \times 10m/s^2) = 3000N$$

Thus this is equivalent to someone weighing about 640 pounds sitting on me.

Tension

Tension is pretty easy to deal with. As we all know, you can attach a rope to something (say a block) and pull. You exert the force on the rope, the rope then exerts the force on the block. The reason it is called tension is that because of the third law, the block also pulls back on the rope with equal and opposite force. Thus as far as the rope is concerned, it is being pulled by both ends. The forces tend to pull the rope apart.

Also note that unless I say otherwise, assume that the rope and any pulleys it may be attached to is massless. All the pulley does is change the direction of the force on the rope (e.g. if you pull down on a rope through a pulley on the ceiling, the resulting force on the block is up of course, not down)

Problem For simplicity, say an elevator of mass 300 kg is hanging from a single cable. What is the tension in the cable if (a) the elevator is moving at a constant velocity? (b) with an upward acceleration of $1.2\,m/s^2$? (c) with a downward acceleration of $1.4m/s^2$?

Answer In all three cases, the downward force is that of gravity,

$$W = (300\,kg)(9.8m/s^2) = 2900N$$

Note this is the magnitude; the direction is always downward. In all three cases, Newton's equation says that

$$ma = \sum F = T - W$$

where T is the tension in the cable. In case (a), the velocity is constant, so there is no acceleration. Thus $T = W = 2900N$. In case (b), the acceleration is positive:

$$T = W + ma = m(g + a) = (300\,kg)(9.8m/s^2 + 1.2m/s^2) = 3300N$$

In case (c), the acceleration is negative:

$$T = W + ma = m(g + a) = (300\,kg)(9.8m/s^2 - 1.4m/s^2) = 2500N$$

The tension increases if you are accelerating an object upwards, it decreases if you are letting it fall downward.

The key thing to remember about tension is that (as long as the rope and pulleys are massless), the only thing the string does is change the *direction* of the force. For the Atwood machine demonstrated in class, the tension on the rope is the same at both blocks. For each of the blocks we can apply Newton's Second Law:

$$\text{Block 1} \qquad m_1 a_1 = T - m_1 g$$

$$\text{Block 2} \qquad m_2 a_2 = T - m_2 g$$

Note that the tension pulls up on both blocks. Note also that in the Atwood machine, the acceleration of the two blocks has the same magnitude, but is in opposite directions. Thus $a_1 = -a_2$. This leaves us with two equations and two unknowns (T and a_1). Adding the two equations gives

$$(m_1 + m_2)a_1 = (m_2 - m_1)g$$

so

$$a_1 = \frac{m_2 - m_1}{m_2 + m_1} g$$

Thus if $m_2 > m_1$, a_1 is positive: the lighter block goes up. We can solve for the tension as well:

$$T = m_1(a_1 + g) = m_1 g\left(\frac{m_2 - m_1}{m_2 + m_1} + 1\right) = g\frac{2m_1 m_2}{m_1 + m_2}$$

Normal Force

When I am standing on the earth, I am not accelerating because the earth is pushing back up on me (Newton's third law). Therefore $\Sigma\vec{F} = 0$ and hence $\vec{a} = 0$. Nevertheless, the weight says how much force being put on my body by gravity (i.e determines if I crack under the pressure). There are two forces on me: gravity and the *normal* force. The first is denoted by $\vec{W}$, while the latter is denoted by $\vec{F_N}$.

In general, you should think of the normal force as the force caused by an object pushing back on another object which is pushing on it. The word "normal" in this context means perpendicular. The reason is the the normal force is always perpendicular to the surface which is causing it. Therefore, to an object on flat ground, the result of gravity pushing down is an equal normal force pointing up. However, if you have an object on a sloping surface, the normal force is not the exact opposite to gravity. It still points "up" instead of "down", but not straight up. Rather, it is perpendicular to the slope.

So let's look at the favorite problems of introductory mechanics: a block on an inclined plane. (If you want to be more dramatic, call it a skier on a very smooth mountain.) I've drawn the force diagram in figure 1.

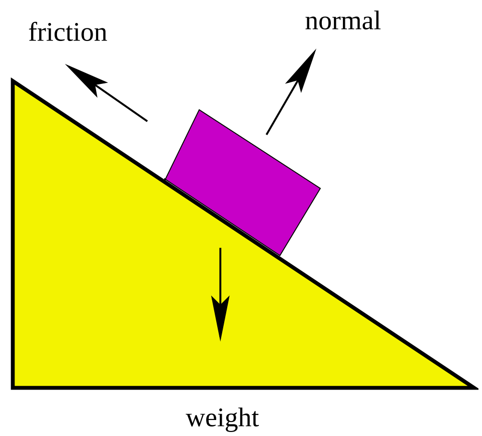

Figure 1: The forces on the *block* on a slope.

Problem Say the 30^o slope in Figure 1 is frictionless, so the only forces are the normal force and gravity. What is the block's acceleration down the slope? What is the normal force, if the mass is $3.0kg$?

Answer This is a vector problem, so we need to divide the problem into coordinates. Instead of the horizontal and vertical coordinates we usually use, for this problem it turns out to be much easier to use coordinates which are parallel and perpendicular to the slope. The forces which act perpendicular to the plane are gravity and the normal force, whereas parallel to the plane there is only gravity. To compute the acceleration and normal force, we need to divide the weight into its components parallel and perpendicular to the slope: the component parallel is $mg\sin(30^o)$, while the component perpendicular is $mg\cos(30^o)$. The normal force is perpendicular to the slope. Thus the only force parallel to the slope is gravity, which is $mg\sin(30^o)$. Thus the acceleration down the slope is

$$\begin{aligned} a_{\|} &= \frac{\Sigma F_{\|}}{m} \\ &= g\sin(30^o) = (9.8m/s^2)(.5) = 4.9m/s^2 \end{aligned}$$

where I used the fact that $\sin(30^o) = 1/2$. Thus we don't even need to know the normal force to compute the acceleration down the slope. To compute it, we use the force equation perpendicular to the slope. Since the block is not leaping off the slope, its acceleration perpendicular to the slope is zero. Therefore the sum of the forces in the perpendicular direction must be zero, so

$$0 = \Sigma F_{\perp} = F_N - mg\cos(30^o)$$

Using the fact that $\cos(30^o) = \sqrt{3}/2 = .866\ldots$, we have

$$F_N = mg\cos(30^o) = (3.0kg)(9.8m/s^2)(.866) = 25N$$

Note that the acceleration down the slope is independent of the mass, but the normal force is not.

Helpful Hint In these sorts of problems it's very easy to mix up the sin with the cos. An easy way to check if you've done it right is to think about the case where the angle is 0 degrees instead of 30: in other words, there is no slope. Then you should have 0 degrees in the above formula instead of 30. Using the fact that $\cos(0^o) = 1$ and $\sin(0^o) = 0$, you check that for no slope, gravity is perpendicular to the plane, as it must be.

Friction

When you are pushing on a surface with your hand, it becomes hard to slide your hand along the surface. The reason, of course, is friction. The harder you push, the harder it is to slide your hand. It might get so difficult to slide it that in fact it might be completely impossible to move it altogether.

The way the frictional force works is a little trickier than the others we've dealt with so far. If the two objects aren't pushing against each other, there is no friction at all. As I said, the harder they're pushed together, the greater the friction. The measure of "The harder they're pushed together" is the normal force. The greater the normal force, the greater the friction. Denoting the frictional force by f and the normal force by F_N, this means that

$$f \propto F_N$$

(Remember that the symbol $\propto$ means "proportional to".) Now it gets even trickier. From our everyday experience, we know that when trying to slide an object along a surface, it is harder to get the object started than it is to keep it moving (once you get it moving, there's usually a lurch forward). Thus there are two different *coefficients of friction*.

Let's see how the frictional force works on objects which are already stopped. This is called static friction. This is a little bit complicated to write down equations for, but the idea is very simple. The complication is that if an object is stopped, and there's nothing trying to push it, there's of course no friction. The friction comes only when you or something tries to push it. Then the friction can push back no harder than you're already pushing it (if it did, the object would move backwards, certainly friction is not capable of doing that). If you increase the force on the object, the force of friction opposing it will get larger and larger, until finally you overcome it. This lets us define the *coefficient of static friction* μ_s in terms of this force $f_{overcome}$:

$$f_{overcome} = \mu_S F_N$$

$f_{overcome}$ is the smallest force which will get the object moving. Your book gives it another perfectly good name: $f_s^{MAX} = f_{overcome}$. The force f_s^{MAX} is the maximal force that friction can apply: the force of static friction always obeys $f_s < f_s^{MAX}$. If you apply more than this, then you overcome friction!

As we saw before, the laws in higher dimensions are the same: one just needs to keep track of all the directions separately. Let's go back to figure 1, but include friction this time. Gravity always points down, towards the center of the earth. The normal force is opposing gravity, but is not directly opposite here. It points perpendicular (or normal) to the plain. If the block is still or if it is moving down the slope (the only possibilities if there are no other forces in the problem), then the frictional force is up the slope.

Statics Problem A block of mass $3.00kg$ is sitting on the slope without moving. The angle the block makes with the horizontal is 30^o. Find the weight, the normal force and the frictional force.

Answer Let's do this problem in x and y components to make it clear that I haven't done anything shady with the coordinates used above. The first thing to note is that since the object is not moving, it is not accelerating. Thus $\vec{a} = (0, 0)$, and by the second law the sum of the forces must also be zero: $\Sigma\vec{F} = (0, 0)$. Now let's look at the three individual forces. The weight is in the down direction, and is of magnitude $W = (3.00kg)(9.80m/s^2) = 29.4N$. Thus

$$\vec{W} = (0, -29.4N)$$

We can't just read off the normal instantly. We need to divide it into components. Using trigonometry, one has

$$\vec{F_N} = (-F_N \sin(30^o), F_N \cos(30^o))$$

Notice that I have used F_N without the vector sign to be the magnitude of the normal force. Similarly, the force of friction is

$$\vec{F_f} = (F_N \cos(30^o), F_N \sin(30^o))$$

So how do we find F_N and F_f? The key is to use the fact that there is no acceleration and that the sum of the forces vanishes. Since force is a vector, this means every component of the force vanishes. Thus as a vector equation we have

$$\begin{aligned} (0, 0) &= \Sigma\vec{F} = \vec{W} + \vec{F_N} + \vec{F_f} \\ &= (0, -29.4N) \ + \ (-F_N \sin(30^o), F_N \cos(30^o)) \ + \ (F_f \cos(30^o), F_f \sin(30^o)) \end{aligned}$$

Let's look at the x component of this equation first. It says that

$$0 = 0 + F_N \sin(30^o) - F_f \cos(30^o)$$

Thus we can rewrite this as

$$F_f = F_N \tan(30^o)$$

This is one equation with two unknowns. To find them both, we need to look at the equation for the y components as well, which is

$$0 = -29.4N + F_N \cos(30^o) + F_f \sin(30^o)$$

Let's check the signs in these equations, because if you mess up any sign, the whole answer will be wrong. In the x component equation, the force of friction is to the left, while the normal force is to the right. In the y component, the force of gravity is down, while the force of the normal and the force of friction are both up. So now, we have two equations and two unknowns. These two are easy to solve. Plug in $F_f = F_N \tan(30^o)$ into the y equation to get

$$29.4N = F_N \cos(30^o) + F_N \tan(30^o) \sin(30^o)$$

To get the cosine and sine, you can use your calculator, or you can remember that $\cos(30^o) = \sqrt{3}/2 = .866\ldots$ and $\sin(30^o) = 1/2$. Either way, you get

$$F_N = 25.4\,N$$

Finally, to get the frictional force, just use the relation

$$F_f = F_N \tan(30^0) = F_N/\sqrt{3} = 14.7\,N$$

I went through that problem with standard x and y coordinates to demonstrate in explicit detail how it works. It's a little bit easier to solve the problem in coordinates parallel and perpendicular to the slope, like we did above. This avoids the two equations, two unknowns, but we can instead solve for the friction and normal force individually. Since the net force is zero, the net force must still be zero for this coordinates as well. The equation for the coordinate perpendicular to the slope involves only gravity and the normal force, namely

$$0 = F_N - 29.4\ \cos(30^o)\,N$$

which lets us recover $F_N = 25.4\,N$. Notice that with these coordinates, we have to divide gravity into components. For the coordinate parallel to the slope, we have

$$0 = F_f - 29.4\ \sin(30^o)\,N$$

and we recover again $F_f = 14.7\,N$. As should be clear from the force diagram, F_f and F_N are perpendicular, and since the net force is zero, their magnitudes must obey

$$W^2 = (F_f)^2 + (F_N)^2$$

This is another good way of checking your results. The first way of doing the problem requires more algebra, but it may be clearer to use x and y coordinates. You should do the problems whichever way feels most natural to you (or do them both ways as a check!).

Note that we did not use the coefficient of static friction in the last problem. The reason is that the coefficient of static friction only gives the maximal friction possible. In the above problem you have no way of knowing what the maximal static friction is, all you know is that there's enough friction to keep the block from sliding down the slope. But here's a problem where you can compute the coefficient.

Problem I slowly tilt the slope upwards. When the slope makes an angle with the horizontal of θ_{slip}, the block starts to slide down the slope. What is μ_s ?

Answer To get μ_s, we need to know the normal force and frictional force at the angle θ. Repeating the arguments in the second way of doing the above problem, but now at an angle of θ, we get

$$F_f = mg\sin(\theta)$$

$$F_N = mg\cos(\theta)$$

The maximal force of friction occurs at the value of θ when the block just starts to slip. This value is called θ_{slip}. Then $F_f^{MAX} = mg\sin(\theta_{slip})$ This value of θ lets us find μ_s, because $F_f^{MAX} = \mu_s F_N$, or

$$mg\sin(\theta_{slip}) = \mu_s mg\cos(\theta_{slip})$$

This means that

$$\mu_s = \frac{\sin(\theta_{slip})}{\cos(\theta_{slip})} = \tan(\theta_{slip})$$