Lecture 7

- Kinetic Friction
- Earthquakes and Newton's Laws

Cutnell+Johnson: 4.9-4.12

Kinetic Friction

Now let's see how friction works on an object which is moving. The constant of proportionality here is called the *coefficient of kinetic friction* μ_k. It is defined by

$$f_k = \mu_k N$$

This coefficient tells you how the friction is related to the normal force. The larger the coefficient, the larger the friction is for a given normal force. Thus a rough surface will generally have a large coefficient than a smooth one. Notice that there are no vectors in the above equation for friction: it applies to the magnitudes of the forces. The direction of the frictional force is always along the surface which perpendicular to the surface which is producing it. In other words, friction is perpendicular to the normal force which is causing it. Moreover, the direction is always opposite to the *motion*, no matter what other forces are being applied. This makes sense when you think about it: friction can never make an object speed up: it always slows it down. Note also that both f_k and N have dimensions of force, so the coefficient of friction has no dimensions - it is just a number (actually, that's why it's called a coefficient).

Problem A puck starts out on the ice with a speed of $20m/s$. The puck weighs $.5\,kg$, and the ice has a coefficient of kinetic friction $\mu_k = .4$. How far does the puck travel before it stops?

Answer First, we need to compute the puck's weight. It's $W = mg = (.5\,kg)(10m/s^2) = 5N$. This force is directed down into the ice (the ice of course is flat). Thus there is a normal force of magnitude $5N$ directed up (perpendicular to the surface of the ice). The force of friction opposes the puck's motion, and has magnitude

$$f_k = \mu_k N = (.4)(5\,N) = 2N$$

A force of $2\,N$ on the puck results in an acceleration of

$$a = \frac{f_k}{m} = \frac{2\,N}{.5kg} = 4m/s^2$$

on the puck in the direction opposite to the motion (so this you can call a deceleration). Now this turns into a problem like we've done before: the puck starts with a speed of $20m/s$ and decelerates at $-4m/s^2$. Thus to compute the time to stop, we use

$$\begin{aligned} v &= v_0 + at \\ 0 &= 20m/s + (-4m/s^2)t \end{aligned}$$

so $t = 5s$. To find the travel distance, use the formula

$$\begin{aligned} x &= v_0 t + at^2/2 \\ x &= (20m/s)(5s) + (-4m/s^2)(5s)^2/2 = 50m \end{aligned}$$

How far would the puck travel if it were twice as heavy (but still started with the same velocity)?

Since the inclined plane is so delightful, let's do one more problem.

Problem Say the coefficient of *kinetic* friction of the block on the slope is $\mu_k = .8$. What is the block's acceleration down the slope once it starts moving (say at $\theta_{slip} = 45^o$)?

Answer Since the block stays on the plane, the acceleration perpendicular to the plane must vanish. Thus the normal force remains the same as in the last problem, so $F_N = mg\cos(45^o) = 20.8\,N$. This means that the force of kinetic friction on the moving block is

$$F_f = \mu_k F_N = (.8)(20.8N) = 16.6N$$

This force is directed up the slope, against the motion. Now the sum of the forces down the slope is not zero. Instead, the acceleration down the slope a is given by

$$\begin{aligned} ma &= F_f - W\sin(45^o) \\ (3kg)a &= 16.6N - 29.4/\sqrt{2}\,N \end{aligned}$$

Solving for a gives $a = -1.36m/s^2$, the acceleration down the plane. You can see the effect of friction by comparing this with the result if there were no friction. With no friction, the acceleration is $a = (29.4/\sqrt{2})/3\ m/s^2 = 6.9m/s^2$. Thus the effect of friction is substantial.

Earthquakes and Newton's Laws

Since Newton's laws are the basis for much of the rest of the class, I'm going to review them here. To make it interesting, let's do it by discussing earthquakes.

Here's how an earthquake works. There are cracks in the earth's crust, called faults. The earth likes to move, at rate of inches/year. The problem is that the earth on the two sides of the fault line isn't necessarily moving at the same speed, or even the same direction. If there weren't friction, this wouldn't be a problem: you would just see your neighbors on the other side of the fault line slide by as the years went by. However, because of friction, the earth on the two sides gets stuck together, and can't move, because the force is not enough to overcome static friction.

So what happens? The earth is still trying to move, but it is stuck. Thus the earth gets compressed and pressure builds up (we'll discuss pressure in more detail later in the class). As the pressure builds, the force on the fault gets greater and greater. Finally, the force gets greater than $\mu_s F_N$, and the whole earth moves. Because μ_k is smaller than μ_s, the kinetic friction can slow down the movement but not prevent it all together. The force built up is enormous, so the earth moves at a rate of feet per second, as opposed to inches per year. This type of motion is called "stick/slip", and should be familiar from everyday life (for example, trying to slide a box across the floor).

Thus this an application of Newton's *Second Law*: the *sum* of the forces is mass times acceleration. Since the sum of the forces built up before an earthquake is enormous, the acceleration is substantial.

The effects of a big earthquake are of course substantial. We read on the news about the buildings that collapse. Let's look at some simpler things which happen, as examples of the Newton's *First Law*.

I lived about 10 miles from the epicenter of the 1994 Los Angeles quake. No buildings collapsed, but one common effect was that lots of chimneys fell (don't ask why one needs chimneys in LA). One thing to note about these chimneys is that they always broke near the roof of the house. This is an example of Newton's first law: the lower part of the chimney is attached to the house, so when the ground moves, the house moves, and the lower part of the chimney moves too. However, the upper part of the chimney sticks over the house. Obeying the first law, it just wants to sit where it is. The only thing holding it to the bottom half is the mortar between the bricks. Mortar is reasonably strong, but over the years cracks develop, there are weak points, and the enormous force on the bottom part breaks the chimney apart. Nobody was underneath these chimneys (it was 4:30 in the morning), but several cars met their maker.

Other simple examples of the first law in an earthquake: the main thing people had broken was TVs, from falling off their stand. The same thing happens to poorly built houses. If the builder is trying to save money, they don't bother to bolt the house to the foundation. In an earthquake, the foundation moves, the house doesn't. (Even if the house doesn't fall off altogether, some houses have to be torn down because they moved too far).

The *third law* of course appears everywhere in earthquake. If some part of the ground pushes on another part, the other part pushes back. All this pushing means the ground starts moving all over the place. Then other faults can get into the act: they've been building up pressure, and the force from the first earthquake is enough to get those faults to slip. Then these push back on the original one, etc, etc. This is the reason for aftershocks (which go on for weeks after a large earthquake).

The same goes for buildings: one part pushes on another, the other pushes back. This creates all sorts of unusual stresses, which is why buildings with a flaw can fall apart.

As a review of Newton's laws, let's do another problem using tension

Problem Three weights are tied onto two ropes and two pulleys. One weight hangs directly below the left pulley, one hangs below the right pulley, while the third weight is tied to both ropes in between the two pulleys. The system is in equilibrium: nothing is moving. The mass of the center weight as $1.00\,kg$, the angle the left rope is tied to the center weight is 30^o from the horizontal, while the angle the right rope makes is 45^o. What are the masses of the two side weights?

Answer For this one you really need to draw a force diagram for each block. For the two side blocks, this is easy. Gravity pulls them down, the tension on that rope pulls it up. Let's call the magnitudes of the two tensions T_1 and T_2. For the center block, there are two ropes pulling up at angles 30^o and 45^o, as well as gravity pulling down. Because the pulleys and the ropes are massless, the tension of ropes 1 and 2 at the center are T_1 and T_2 respectively. Now we use the fact that $\vec{a} = 0$ and so $\sum \vec{F} = 0$. Note that this is a vector relation: the forces in the x and y directions vanish on *all* the blocks. Let's look first at the left block. The tension points straight up, gravity straight down. Since the acceleration vanishes, the sum of the forces is zero, so

$$0 = T_1 - m_1 g$$

where m_1 is the unknown mass of the left block. Likewise, for the right block

$$0 = T_2 - m_2 g$$

This now lets us figure out what's happening to the center weight. Let's look first at the horizontal forces. These come only from the two ropes, because gravity's force is always down. We need the horizontal components, which are $T_1 \cos(30^o)$ to the left, and $T_2 \cos(45^o)$ to the right. Because the horizontal acceleration is zero, the sum of the horizontal forces is zero, so

$$0 = T_2 \cos(45^o) - T_1 \cos(30^o) \qquad (1)$$

The forces on the center block in the vertical direction are the vertical components of the two tensions pointing up, while gravity points down. The acceleration in the vertical direction is also zero, so the sum of the vertical forces is also zero, giving

$$0 = T_1 \sin(30^o) + T_2 \sin(45^o) - m_{center} g \qquad (2)$$

Since $T_1 = m_1 g$, $T_2 = m_2 g$, and $m_{center} = 1.00kg$, we have two equations, two unknowns (m_1 and m_2). The first equation (1) becomes

$$m_2 = \frac{\sqrt{6}}{2} m_1$$

Substituting this into the second equation gives

$$0 = m_1 \frac{1}{2} + m_1 \frac{\sqrt{6}}{2} \frac{1}{\sqrt{2}} - (1.00\,kg)$$

Notice that the factor of g appears in all terms so I can divide it out.