Lecture 9

- Conservation of Energy
- Kinetic Energy
- Work

Cutnell+Johnson: 6.8, 6.1-6.2

Conservation of Energy

One of the most fundamental principles of physics is that energy is conserved. And as you know, when something is really important we call it a law. And the law of conservation of energy is simple:

> Energy is always conserved. It may change forms, but the total amount of energy is a closed system is conserved.

So the tricky part is that energy can change forms. There are many different forms of energy. Two we will deal with in this chapter are

- Kinetic Energy (Energy of Motion)
- Gravitational Potential Energy

Others we will deal with later in the course are

- Heat
- Electromagnetic energy (this includes light)

One form of energy we will discuss frequently qualitatively, but not quantitatively is

- Chemical energy

I say we will frequently discuss it because chemical energy is the kind of energy your body makes: various chemical reactions happen inside your body which release energy, which you use to do exciting things like push blocks up an inclined plane. I say not quantitatively, because we will not discuss how these chemical reactions work, I will just say "you push a block up an inclined plane". Finally, there's one form we probably won't discuss any further in this context, but it's interesting, so I thought I'd mention it:

- Mass

The energy in a given mass is given by Einstein's famous formula $E = mc^2$, where c is the speed of light. A nuclear reactor, for example, converts mass energy to heat . The sun also converts mass to other forms of energy (light and heat), but through a different process called fusion.

So you can make diagrams which govern the energy conversion. For example, you can have

$$\text{mass} \longrightarrow \text{heat} \longrightarrow \text{chemical} \longrightarrow \text{kinetic} \longrightarrow \text{electrical} \longrightarrow \text{light}$$

This describes a nuclear reactor, which makes heat, which boils water, which turns turbines, which make electricity, which you use to power a light. In Virginia, it's even more elaborate: the power companies here use nuclear power from Illinois at night to pump a lake up a hill (gravitational potential energy). The lake is let down during the day, which turns turbines, etc. You can come up with all sorts of examples, i.e. a battery converts chemical energy to electrical energy.

Kinetic Energy

A moving object has energy. This is called *kinetic energy.* It shouldn't be surprising that motion is a form of energy, because we can easily imagine ways of converting motion to other forms of energy. For example, if friction slows down a moving object, the object gets hotter, thus converting kinetic energy to heat. An example of this is a meteorite burning up in the atmosphere: the heat to do this occurs because of air friction.

There is a simple formula which give the energy of a moving object. We won't derive the formula, but we will use it frequently. An object of mass m and speed v has kinetic energy

$$KE = \frac{1}{2}mv^2$$

Notice that an object does not need to be accelerating to have kinetic energy. In fact, if the acceleration causes a change in speed, then it changes the kinetic energy. (Recall that an acceleration means a change in velocity, not necessarily a change in speed. Centripetal acceleration is an example of an acceleration which does not change the speed.)

The MKS unit of energy is easily found from the above formula. It is a kgm^2/s^2. Since this is unwieldy, we define a special unit, called the *Joule*, abbreviated by J. Thus

$$1\,J = 1\frac{kg\,m^2}{s^2}$$

Notice that it is not the same as a Newton, the unit of force, which is $1\,N = 1\,kgm/s^2$. Thus

$$1\,J = 1\,N\,m$$

i.e. one Joule is one Newton-meter. This formula will be relevant in the next section when I discuss work. There are a variety of other units of energy. Other metric ones are *calories*, *ergs* and *kilowatt-hours*. Some English units are *British thermal units (BTU)* and *foot-pounds*. We'll get back to some of these later in the course.

Work

Like many of the other things we have studied, we have some intuition into work. Work is closely related to energy: we have the idea that we do work when we expend energy. That's close, but the definition of work we use in physics is a little narrower. If forces are acting on an object which cause it to change its speed, then the total work done on the object is the change in kinetic energy, namely

$$W = \Delta\ KE$$

The units for work are therefore the same as those of energy. So notice that if the kinetic energy does not change, then no work is being done. Thus by this definition, when I twirl an object in a circle, I am not doing any work. Similarly, if I am pushing hard against a wall without the wall moving, I am not doing any work by this definition.

I am doing work when I cause an object to change its speed. This means that the amount of work I do is related the amount of force I put on an object. There is a useful formula which relates work to the force. Say there is a net force $\vec{F}$ on an object, which as a result moves some with some displacement $\vec{d}$. (I should write ΣF for the net forces but in this lecture I'll omit the Σ – today F will always mean the net force.) Then the work done on the object is

$$W = Fd\cos\theta$$

where θ is the angle between $\vec{F}$ and $\vec{d}$, and F and d are the corresponding magnitudes.

Let's first look at some special cases.

1. If the force is applied in the same direction that the object is moving, then $\theta = 0$, and $\cos(0^o) = 1$. Then simply $W = Fd$.
2. If the object does not move, then $\vec{d} = 0$. In this case, $W = 0$. This agrees with the earlier formula, that work is the change in kinetic energy: obviously if an object is not moving, then its kinetic energy is not changing.
3. In the case of centripetal acceleration, the object is moving, and there is being a force applied. However, there is still no work being done by this definition. This is obvious from the first formula: because the speed remains the same (even though the velocity is changing), the kinetic energy does not change and there is no work done. This also follows from the second formula. The reason is that the force causing the centripetal acceleration is towards the inside, i.e. radial. However, the displacement is the change in position, and so that is tangential to the circle. Thus the force and the displacement are always perpendicular to each other, so the angle θ between them is 90^o: Since $\cos(90^o) = 0$, the work is zero.

The reason we introduce work is that it can be useful to combine the two formulas. For example,

Problem Say a hockey player hits a $.50\,kg$ puck with a force of $100N$ over a distance of $.30m$. What speed does the puck leave the stick with (neglecting friction)?

Answer When you hit a puck with a stick, you are hitting it horizontally, the same direction the puck is moving. Therefore the angle between the force and the displacement is zero, and so $\cos\theta = 1$. The work done on the puck is therefore

$$W = (100\,N)(.3m) = 30\,J$$

Notice that a joule is a newton-meter. The work is also the change in kinetic energy. The initial kinetic energy is zero, because the puck starts at zero velocity. The change in kinetic energy is therefore

$$\Delta KE = \frac{1}{2}mv^2$$

where v is the final speed. Comparing the two, we see that

$$v^2 = \frac{2W}{m} = \frac{2 \times 30\,N}{.5\,kg} = 120m^2/s^2$$

Taking the square root gives $v = 11m/s$.

You could have solved this problems using the earlier techniques we discussed, but it probably was easier this way. However, we can see how the two methods are the same. Recall the formula

$$v^2 - v_0^2 = 2ax$$

Now multiply both sides by $m/2$. We now have

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = max$$

But now we use the fact that $KE = mv^2/2$, and $F = ma$ to write

$$\Delta KE = Fx$$

or $W = \Delta KE$!

Problem

I sweep the ice with a $1\,kg$ broom, using a force of $20N$ at an angle of 45^o with the horizontal. How far do I have to push it to give it a speed of $4m/s$, neglecting friction?

Answer

The change in kinetic energy of the broom is

$$\Delta KE = \frac{1}{2}mv^2 = \frac{1}{2}1\,kg(4m/s)^2 = 8J$$

The angle between the force of my pushing is and the displacement is 45^o, so the work in the frictionless case is

$$W = Fd\cos(45^o) = (20\,N)d\frac{1}{\sqrt{2}}$$

Since $W = \Delta KE$, we have

$$d_{frictionless} = \frac{\Delta KE}{F\cos(45^o)} = \frac{8\,J}{20/\sqrt{2}\ N} = .57\,m = 57cm$$