

Physics 252 – Modern Physics
Homework Assignment #1

Due Friday, January 26, 2007 at the beginning of class.
Late assignments get no credit.

Reading: Feynman, chapter 1 and section 2.1; Fowler, “Wave Equations for Photons and Electrons”, “Probabilities, Amplitudes and Probability Amplitudes”, “Rays and Particles”

1. In Fowler’s notes you will find a derivation of the fact that each component of the electric field and magnetic field in a vacuum satisfies the wave equation. Calling any one of these components ψ , one has in one spatial dimension

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

Verify that $e^{ikx-i\omega t}$ is a solution of this equation when k and ω obey a certain condition. What is this condition? How do k and ω relate to frequency and wavelength?

2. In more than one dimension, the wave equation is

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

where in three dimensions

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

What is the solution to this equation which corresponds to a wave moving in the direction $\hat{x} + \hat{y}$? ($\hat{x} + \hat{y}$ is the vector notation for the direction 45 degrees from the x axis, 45 degrees from the y axis, and 90 degrees from the z axis.) What is the most general solution to the wave equation corresponding to the wave moving in an arbitrary direction \hat{r} ? (Hint: try *multiplying* solutions of the 1d wave equation together.)

3. Estimate the wavelength of a typical bullet in flight. Your answer should be a number with units, not a formula.
4. Neutrons at room temperature have a kinetic energy of around $.02 eV$. What is their wavelength?
5. Say we have a wave in a one-dimensional box of length L . The amplitude of the wave vanishes at the edges of the box. What are the possible wavelengths?
6. Say we trap an electron in a one-dimensional box. This means that if you think of the electron as a wave, the amplitude of the wave must vanish at the edge of the box. What is the electron’s lowest possible velocity if the length of the box is $1 mm$? What if the box were atomic size?