Physics 252 – **Modern Physics**

**Homework Assignment #3**

Due Friday February 9, 2007 at the beginning of class.

Reading: Feynman, sections 2.4-5 and chapter 3; Fowler, "The Bohr Atom", "From the Bohr Atom to De Broglie's Waves", "Spectra"

1. It was found experimentally that all the light emitted from various atoms has a wavelength which fits into the formula
$$\frac{1}{\lambda_{mn}} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{1}$$
where $m$ and $n$ are positive integers with $m > n$, and $R$ depends only on the kind of atom. Show that this formula can be explained by requiring 1) light occurs in quanta (photons) and 2) the electron can only be in energy levels with a fixed $E_n$, with $n$ an integer. What specific formula for $E_n$ explains the formula (1)?

2. In this and the next two problems, you'll "derive" the formula for $E_n$ in the simplified model of an atom, where electrons move in circular orbits of fixed radius (the "Bohr atom"). In this problem, show how angular momentum is quantized. To do this, first find the quantization condition for the angular component of the momentum by demanding that for a given orbit at radius $a$, there be an integer number of wavelengths around the orbit. Then use $p = h/\lambda$ to find the quantization condition for the angular component of the momentum. Then turn this into a condition for the angular momentum. (Hint: you will get an answer $L_n$ that depends on an integer $n$, and of course on $\hbar$, but will be independent of $a$.)

3. We computed the size of the atom by using the uncertainty principle to estimate the momentum of the electron. Using angular momentum quantization, you can do better. First, relate the angular momentum to the kinetic energy classically. Then, use this to write the total energy (including the electrostatic potential) of the electron in the hydrogen atom in terms of $n$, $a$, and fundamental constants. Then find the radius $a_n$ of the electron with angular momentum $L_n$, by finding the minimum of this energy.

4. Finally, use your answer for $a_n$ to get $E_n$ for the hydrogen atom. What is $R$ for hydrogen in terms of fundamental constants? What is its numerical value?

5. In the last homework you derived the energy uncertainty relation $\Delta E \Delta t \approx \hbar$. For an atom in an unstable state, $\Delta t$ is the average lifetime of that state. The lifetime of the $n = 2$ excited state in hydrogen is about $1.4 \times 10^{-12} s$. The photon emitted when this electron drops back to the $n = 1$ state will thus have a slight uncertainty in energy. This will result in a range of frequencies for this photon, which is known as "line broadening". What is this range here?

6. A certain crystal has a planar spacing of $0.32\, nm$. What neutron energies are necessary to observe up to three interference maxima for neutron scattering? (Hint: Don't use $E = h\nu$, because the neutrons are massive.)