

## Physics 252 – Modern Physics

### Homework Assignment #5

Due Friday, February 23, 2007 at the beginning of class.

The first midterm will be on Wednesday, February 28. There will be no notes, no book, no calculators permitted. It will cover material up to and including that on this problem set. Patrick will conduct a review session on Monday night, Feb. 26th, time and place to be announced. There will be no section meetings that week, and class Friday, March 2 is cancelled.

Reading: Feynman, chapter 4; Fowler, “Blackbody radiation”, “The Bohr atom”

Integrals you may need:

$$\int_0^{\infty} dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15} \quad \int_0^{\infty} dx \frac{x^2}{e^x - 1} = 2\zeta(3) = 2 \left( 1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \dots \right) = 2.404113\dots$$

Possibly-useful trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1, \quad 2 \cos(a) \cos(b) = \cos(a + b) + \cos(a - b) \quad \sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$

1. Rerun the argument for the Bohr atom in the case of a general atom, not just hydrogen (i.e. assume the nucleus has charge  $Ze$ , not  $e$ .) You don't need to give all the details again, just indicate how and why the different charge changes the final answer for the energy levels.
2. Consider a particle of mass  $m$  in a one-dimensional box of length  $L$ . Write down the most general amplitude which allows it to be in a linear combination of the states with the two lowest energies. Work out the normalization. How many independent parameters are left? In terms of your parameters, work out what the probability is that it is in the lowest-energy state?
3. The sun's temperature is about  $6000 K$ , its radius  $700000 km$ . How much power is it radiating? Assuming no dissipation between here and the sun, how much of this hits the earth?
4. Derive the formula for the total energy density of the black body given in class. Argue why the Stefan-Boltzmann law is true (don't worry about the factor of 4).
5. Derive the total energy density for a black body in a two-dimensional world. (What changes in the computation is the “density of states”, how many states there are with a given energy.)
6. For blackbody radiation, derive an equation whose solution yields  $\omega_{peak}$ . You do not need to solve this equation, but instead determine how  $\omega_{peak}$  depends on the temperature, e.g. if we double  $T$ , how does  $\omega_{peak}$  change?