

Physics 252 – Modern Physics

Homework Assignment #7

Due Friday, March 23, 2007 at the beginning of class.

The Hoxton lecture will be at 7:30 p.m. next Thursday (the 22nd) in the Chemistry Building, Room 402. The Speaker is John Mather, who won the Nobel Prize last year for discovering anisotropies in the cosmic microwave background.

Reading: Feynman, chapter 7, 8.6, 9.1-9.5

1. Consider two particles with spin  $1/2$ . Say they are in the “spin singlet” state

$$|s.s.\rangle = \frac{1}{\sqrt{2}}|+Z, -Z\rangle - \frac{1}{\sqrt{2}}|-Z, +Z\rangle$$

where  $|+Z, -Z\rangle$  is the state where the first spin has  $S_z^{(1)} = \hbar/2$  and the second has  $S_z^{(2)} = -\hbar/2$ , and  $|-Z, +Z\rangle$  is the reverse. This state  $|s.s.\rangle$  has total  $S_z = S_z^{(1)} + S_z^{(2)} = 0$ . Show that in this spin singlet state, the overall  $S_x = S_x^{(1)} + S_x^{(2)} = 0$  and that  $\vec{S} \cdot \vec{S} = 0$ . **Hint:** The two-particle state  $|+Z, -Z\rangle$  should be treated as the *product*  $|+Z\rangle|-Z\rangle$  of two one-particle systems.

2. Consider a spin- $1/2$  particle with magnetic dipole moment  $\mu$  in a magnetic field  $B$  in the  $z$  direction. Fix the zero of energy so that the particle has zero energy when  $B = 0$ . Write down an arbitrary state of this system in terms of the stationary states. Say this system has a definite value of  $S_x = \hbar/2$  at time  $t = 0$ . What is the state at some later time  $t$ ? What is the probability that this system has  $S_x = -\hbar/2$  at some later time  $t$ ?
3. Consider the ammonia molecule as a two-state system, as discussed in class and the book. Derive the energies of the stationary states for (a) tunneling without an electric field and (b) and a constant electric field without tunneling.
4. Look ahead in the book, and give two more examples of two-state systems. Explain when and why the two-state system is a good approximation for the systems you chose. Also say whether the two states in these systems are symmetric (like the ammonia molecule without an electric field), or different (ammonia with an electric field).
5. This and the next problem follow section 7.4 of Feynman to check that the classical limit of quantum mechanics agrees with the usual computations. Consider a particle moving through a region of variable potential, as illustrated in Fig. 7-7. Classically, compute the deflection of the particle  $\delta\theta$ . Quantum mechanically, this deflection occurs because of the uncertainty principle: a particle is spread out in space, and the probability amplitude in different potentials is affected differently. Compute the difference in  $p$  between two tracks a distance  $D$  apart, so that the potential difference  $\Delta V$  between the two tracks is  $D\partial V/\partial y$ .
6. After the wave travels a distance  $w$ , the phase in the amplitude of the track in a lower potential is different from that in the higher track. Compute this difference in phase, and then the difference in position of the peaks of the two waves. Use this to compute  $\delta\theta$ , and explain in which limit the classical and quantum results agree.