Physics 252 – **Modern Physics**

Homework Assignment #8

Due Friday, March 30, 2007 at the beginning of class.
The second midterm will probably be on Friday, April 13; let me know if this will cause problems.

Reading: Feynman, chapter 7; Fowler, "Electron in a box", "Finite square well".

1. Find the explicit stationary states for the ammonia molecule in a constant electric field. Check that your answers agree with those given in class in the limit $\mathcal{E} \to 0$ as well as in the limit $A \to 0$.

2. The remainder of this problem set is devoted to the quantization of the energy of a particle in a one-dimensional box where one side is of finite height. The particle of mass m in a potential which is $V = \infty$ for $x < 0$, $V = 0$ for $0 < x < L$, and $V = V_0$ for $x > L$. Consider a stationary state with energy greater than zero but less than V_0. For $x < 0$, the amplitude is zero. For $0 < x < L$, the amplitude will depend on x as
$$Ae^{ipx/\hbar} + Be^{-ipx/\hbar}$$
For $x > L$, the amplitude is
$$Ce^{-p'x/\hbar}$$
Why is there no $De^{+p'x/\hbar}$ in the latter? By using conservation of energy, relate p to p'.

3. What is the probability the particle is in the classically-forbidden region? You can leave your answer in terms of C and p', but evaluate any integrals involved.

4. Relate A to B by requiring the amplitude be continuous at $x = 0$. In other words, match the expression for $x < 0$ to the one for $0 < x < L$ at $x = 0$. Then relate C to these by demanding that the amplitude be continuous at $x = L$.

5. There are 5 unknowns A, B, C, p, p', with so far three equations relating them. Find a fourth by requiring that at $x = L$, the *first derivatives* of the two amplitudes match. Combine this with the above to eliminate the other unknowns to find a single equation for p. (A fifth equation can be found be demanding that the probability amplitude be normalized properly, but this is not needed to find the equation for p.) Make this equation look nicer by rewriting it in terms of the dimensionless parameters $\widetilde{V} = \frac{2mL^2V_0}{\hbar^2}$ and $\widetilde{p} = \frac{pL}{\hbar}$.

6. The previous equation for the unknown $\widetilde{p}$ can't be solved analytically, but a lot can be learned about it without having to solve it numerically. First check that this equation gives the right answer in the $\widetilde{V} \to \infty$ limit, where the box walls are infinitely high. Now sketch each of the two sides as a function of $\widetilde{p}$; when the two intersect, you have a solution. Show that when $\widetilde{V} = 0$, there are no solutions. As $\widetilde{V}$ is increased, more and more solutions (intersections) occur; explain why this is so. Find the values of $\widetilde{V}$ where a new solution of this equation exists and the number of levels increases by one.