Physics 252 – **Modern Physics**

Homework Assignment #9

Due Friday, April 6, 2007 at the beginning of class.
NOTE: two sides
Test on Friday, April 13.

Reading: Feynman, 18.3, 11.4, 13.1-13.2, 14.1-14.6

1. Consider the finite-depth well you studied in detail on the last homework. Rewrite E/V_0 in terms of the dimensionless variables by using the energy-momentum relation $E = p^2/2m$. Numerically, find a solution for $\widetilde{p}$ and thus E/V_0 when $\widetilde{V} = 6$. Sketch the probability as a function of x for the lowest-energy state.

2. Check explicitly that $\psi(x) = Ae^{i(px-Et)/\hbar}$ satisfies the one-dimensional Schrödinger equation when the potential $V(x)$ is independent of x, as long as p, E and V obey a certain condition. What is this condition?

3. Show explicitly that the three probability amplitudes given in Feynman equations (19.28), (19.29) and (19.30) are spherically-symmetric solutions of the Schrodinger equation in three dimensions. What are their energies?

4. In section 11.4 of Feynman you will find a discussion of linear polarization of photons. Use this to write the $S_z = 0$ state
$$\frac{1}{\sqrt{2}}|RR\rangle - \frac{1}{\sqrt{2}}|LL\rangle$$
in terms of linearly polarized states. Be sure to say which axes your linearly polarized states are defined with respect to. How does the result change if you rotate your x and y axes?

5. Consider the Michelson-Morley experiment. Pretend that light moves with a velocity c with respect to some fixed frame (the frame with the "ether at rest"), so in different frames one would measure different speeds of light. The Michelson-Morley apparatus is moving at a velocity v in a direction along one of the two arms. Then compute the time it takes the light to travel down that arm and back. Do the same computation for the arm perpendicular to it. What is the difference in the two times? The Michelson-Morley experiment actually measures no difference in times. What is Einstein's theoretical explanation of this fact?

   Question 6 on other side!

6. Consider three spin-1/2 particles sent off at 120 degrees to one another, and three detectors which can measure one component of the spin. In quantum mechanics the system can be in a state such that:

   (a) If two of the detectors measure S_y, and the other S_x, then one always measures an odd number of up spins.

   (b) If all three measure S_x, then one always measures an even number of up spins.

   Thus in this state knowing the results for two detectors automatically determines the result for the third. This is analogous to the decay of spin-0 positronium: because the final photons are in the state $|LL\rangle - |RR\rangle$, measuring the polarization of one of the photons tells us what the other one must be.

   The purpose of this problem is to show that there is no way to construct a system with properties (a) and (b) if we know both S_x and S_y at the same time. Only systems which obey a "spin uncertainty principle" can obey properties (a) and (b).

   To show this, assume that we can know both S_x and S_y at the same time. First consider the situation where detector D_1 is measuring S_x, and the other two S_y. There are four possible ways the number of up spins can be odd: in a table, they are

	D_1	D_2	D_3
x	$\uparrow$		
y		$\uparrow$	$\uparrow$

	D_1	D_2	D_3
x	$\uparrow$		
y		$\downarrow$	$\downarrow$

	D_1	D_2	D_3
x	$\downarrow$		
y		$\uparrow$	$\downarrow$

	D_1	D_2	D_3
x	$\downarrow$		
y		$\downarrow$	$\uparrow$

   An empty box means that for this particular measurement, it doesn't matter what that component is – it's not being measured.

   Now this isn't enough. We've said that if *any* one of the three detectors measures S_x, we must get an odd number of up spins when the other two detectors measure S_y. So now take what's written above, and fill in the boxes to assure that if D_2 were measuring S_x, and D_1 and D_3 measuring S_y, we would still get an odd number of up spins. You should get 8 possibilities (with one box still left unfilled: the S_x component of D_3). Fill this box by demanding that if D_3 were measuring S_x and the other two S_y, we would still get an odd number of up spins. You should get 8 possibilities for this, with all boxes filled.

   So this sounds OK. However, we haven't yet studied the other experiment, where all three detectors are measuring S_x. Here we should measure an *even* number of up spins. Check now that *none* of the 8 possibilities you found above have an even number of S_x spins up.

   This is just like Bell's theorem! There is an experimental difference between quantum mechanics and a classical theory with hidden variables. For more details on this particular problem (in particular, to see the quantum-mechanical state which satisfies properties (a) and (b)), see N.D. Mermin, "Quantum Mysteries Revisited", American Journal of Physics vol. 58 (1990), page 731.