Modern Physics notes
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**Lecture 34**

- General Relativity

- Born, chapter III (most of which should be review for you), chapter VII
- Fowler, "Remarks on General Relativity"
- Ashby on General relativity and the GPS, www.phys.lsu.edu/mog/mog9/node9.html

## No absolute acceleration

Everything we did in special relativity involved inertial frames: ones which moved at constant velocities with respect to each other. All this implied that there was some way of distinguishing an accelerating frame from an inertial frame. This, for example, was crucial in understanding the twin paradox. In that case, one can indeed distinguish the two, because the twin in the rocket needs to apply a force (fire the rocket engines) in order to switch inertial frames.

However, when it comes to worrying about forces from gravity, things are subtler. In particular, if we are to distinguish inertial frames from accelerating ones, then this implies the existence of an "absolute rest frame", a sort of gravitational ether. Born gives a nice example of this. Consider two planets, extremely far from each other, so that the gravity of one can't possible affect the other. Say they are both rotating at different speeds around the axis which connects one to the other. Now each will see the other one is an accelerating frame, while I'm at rest.

So who's right? Well, say one planet is more oblate than the other (fatter at the middle, because of centrifugal forces). Then one might say that the more oblate planet is in the accelerating frame. But this then implies the existence of an absolute rest frame: the more the planet accelerates with respect to the absolute frame, the more oblate it gets. This is the only explanation, because the planets are so far apart that gravity can't possibly explain the oblateness.

This is logically consistent: one could postulate the existence of an absolute rest frame. If you're moving at a constant velocity with respect to this, you're in an inertial frame. If you're accelerating with respect to this frame, you're not. The amount of oblateness depends on your acceleration with respect to the absolute rest frame. But since Einstein demolished the idea of an absolute time frame or absolute space frame (i.e. the electromagnetic ether), the thought of an absolute rest frame was fairly suspicious.

Moreover, just like with the electromagnetic ether, there was no experimental evidence for this absolute frame. The reason is that there's no way of just isolating two planets like this. You can look up to the sky and see a frame: that of all the stars. Even though the stars are moving around with respect to each other, it's so far away that we can't really tell over any appreciable time. So they look fixed in space to us. This means we can measure acceleration *relative* to this "fixed" frame. So we don't need the idea of an absolute frame to explain the oblateness, just a relative statement: the more a planet is accelerating with respect to the "fixed" stars, the more oblate it is.

Junking the idea of an absolute rest frame leads to a huge extension of our principle of relativity: we say now that the laws of physics must involve only relative positions and motions. Thus there is no distinguished set of inertial frames: we can only talk of relative accelerations, not absolute ones.

**The Equivalence Principle**

This "general" relativity doesn't change anything about E+M. Nothing in there really requires the existence of an absolute rest frame (indeed we got rid of the idea of an ether). But for gravity, this general relativity has profound consequences.

The reason is because of something which always seemed a coincidence, but which in fact turns out to be a fundamental fact of nature. This is the equivalence between inertial and gravitational mass. No one ever points this out to you when you first learn this, but mass appears in two different ways. The first is in $\sum F = ma$ (or in the special-relativistic generalization). The second is in the force due to gravity, either $F_{grav} = mg$ (or the more general $GMm/r^2$). The fact that for every single body in the universe seems to have the same $m$ in $F = ma$ and $F = mg$ is the equivalence of inertial and gravitational masses.

First let's consider the case of a constant gravitational field. For example, say you stay right at the earth's surface, so $F_{grav} = mg$.) The principle of equivalence means that if there are no other forces in the problem, you can't measure $m$. This is Galileo's famous experiment: all bodies fall with the same acceleration, independent of mass. But our principle of general relativity means that $g$ is a relative quantity as well: it is the acceleration with respect to the surface of the earth.

So far that doesn't sound so weird. But now say I'm in free fall. Then I don't feel the effect of gravity: if I'm on a scale which is also in free fall, the scale registers zero. So there are two ways of viewing the situation. People on the earth's surface would say that there's a gravitational field, and I'm accelerating because of it. I, on the other hand, would say that there's no gravity, and that for some reason the earth is accelerating toward me. Here's the profound consequence of general relativity: you get the same physics in a uniformly accelerating frame as you do in a frame without acceleration in the absence of gravity. If I do an experiment (with the lab in free fall too), I get the same results as someone would get in a frame in the middle of outer space, where there is no gravity.

So let me reiterate: a frame in free fall is just like an inertial frame (without gravity)! In fact, there is no way of distinguishing experimentally between which is gravity and which is acceleration. For example, say you're in a closed box resting on some platform. Say I then lower the platform with an acceleration $a$ even greater than that of gravity, so $|a| > g$. But in the box, all you know is that all of a sudden you're *rising* with an acceleration $a - g$. You have no way of telling whether or not I'm pulling the box downward, or a large planet is now above the box, creating a gravitational field which is larger than the earth's.

Because of the principle of equivalence, you can effectively cancel the effect of gravity by going to an accelerating frame. Or you can make it greater, it's up to you. (This is only true for uniform gravity: when the gravitational field depends on space, changing frames only allows you to cancel gravity in any one region where it is constant.) Only because of the principle of equivalence can you do this: because different particles have different charge-to-mass ratios, you can never transform to a frame where all electromagnetism cancels.

This means gravity bends light. I'm in free fall, and shine a flashlight. I'm a perfectly good inertial frame, and say that the light goes in a straight line. On the ground, the light's trajectory will look curved. The only way of understanding the curving with respect to the earth is to note that I see no gravity, but on the ground they do. So in my frame, there's no gravity, light travels in straight lines. On the ground they say there is gravity, so gravity must bend light!

This bending was observed for the first time during a solar eclipse in 1919 by stars very close to the sun. Their light is bent as it passes around the sun. The eclipse doesn't matter for the bending – it's just necessary to see the stars.

### Gravity affects clocks

An experiment to test this idea was done in the early '60s by Pound and Rebka in a tower 20 feet from where my office was as a graduate student. First consider light shined *downward* in a freely falling elevator of height $h$. Inside the elevator, we're a happy inertial frame. We say it takes time $t = h/c$ to hit the bottom. We also say that there's no Doppler shift of the frequency of the light.

But how does this look from the ground? Say the light beam was emitted just as the elevator was released into free fall (i.e. at zero velocity). By the time the light hits the bottom of the elevator, it is accelerated to some velocity $v$. Since light travels so fast, the elevator isn't traveling very fast when the light hits the bottom, so $v$ is pretty small, and we can use non-relativistic formulas for this (but not the light!). We thus simply have $v = gt = gh/c$.

Now let's see what this does to the frequency of the light. We know that even without special relativity, observers moving at different velocities measure different frequencies. (This is the reason the pitch of an ambulance changes as it passes you – it doesn't change if you're on the ambulance). This is called the Doppler shift, and for small relative velocity $v$ it is easy to show that the frequency shifts from $f$ to $f(1 + v/c)$ (it goes up heading toward you, down away from you). There are relativistic corrections, but these are negligible here.

Now back to our experiment. In the freely-falling elevator, we're inertial and measure the same frequency $f$ at top and bottom. Now to the earth frame. When the light beam is emitted, the elevator is at rest, so earth and elevator agree the frequency is $f$. But when it hits the bottom, the elevator is moving at velocity $v = gh/c$ with respect to the earth, so earth and elevator must measure different frequencies. In the elevator, we know that the frequency is still $f$, so on the ground the frequency

$$f' = f(1 + v/c) = f(1 + gh/c^2)$$

On the earth, we interpret this as meaning that not only does gravity bend light, but changes its frequency as well.

If you start the light beam at the bottom, then the frequency measured on earth decreases – it is shifted to the red. Thus light escaping from a very massive star undergoes a *gravitational red shift*. This can be observed as well, by measuring the spectral lines from these stars and then comparing to those on earth. but Pound and Rebka measured this effect directly in the tower by my office. Note that with the $\sim 20\,m$ tower in a four-story building, this requires measuring a variation in frequency to about 1 part in $10^{15}$. Now, using atomic clocks, this effect is easily measured: the standard clock in Boulder, CO (5400 feet) runs 6 microseconds a year faster than the clock at sea level in Greenwich, England. (Atomic clocks have an accuracy of about a microsecond a year.)

So now forget the elevator – we've shown that light in a gravitational field shifts its frequency. This means that clocks at different altitudes measure time differently differently in the presence of gravity! This is like time dilation, except here nothing needs to be moving – the gravitational field does it. The time between ticks $\Delta t$ is $1/f$, so

$$\frac{\Delta t_{sat}}{\Delta t_{earth}} = \frac{f_{earth}}{f_{sat}} \approx (1 + gh/c^2)$$

for a satellite at height $h$. The earth measures a shorter time than does the satellite.

There is even a practical application of this result: the GPS consists of satellites in orbit with atomic clocks. Your GPS device gets signals from multiple satellites, each encoded with the information of what time and at what position the signal was sent. Since we know the speed of light, you can then figure out your exact position if there are at least three satellites. The speed of light is about one foot per nanosecond, so an error of one nanosecond is about one foot. On the homework, you'll work out by how much the clocks on the satellite change relative to the earth every day. The time you get from the clock thus must be corrected to compensate for this. It also must be corrected for the time dilation due to special relativity (the motion of the satellite – they go around the earth every twelve hours).