Modern Physics notes – Spring 2007

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Lecture 10

- Identical particles

- Feynman, 3.2, 3.4, 4.1

Identical particles

All the experiments we've discussed so far basically involve single particles: the electron in the double-slit experiment is interfering with itself. So when you have a system with multiple particles, even more phenomena can happen. Here we'll discuss a very striking consequence of quantum mechanics: particles which are identical have a novel kind of interference with each other.

First, let's consider a scattering experiment, as shown in Fig. 3-7 of the book. Say you have just one detector D, which can measure any kind of particle you're using in the experiment. In the center of mass frame (meaning that the total momentum is zero), the particles scatter in opposite directions. In our new notation, we can characterize a final state by $\langle\alpha,\beta|$, where α is the angle particle 1 goes into, and β is the angle particle 2 goes into. Then the two states where the detector registers a particle are $\langle\theta,\pi-\theta|$, and $\langle\pi-\theta,\theta|$. Since we have two particles now, we need two sources, and can label our initial state as $|s_1,s_2\rangle$. Then let $f(\theta)$ be the probability amplitude for having particle 1 scatter at an angle θ from a given initial state $|s_1,s_2\rangle$, i.e.

$$f(\theta)\equiv\langle\theta,\pi-\theta|s_1,s_2\rangle$$

In words, this means that the probability amplitude that you see particle 1 in detector D at angle θ is $f(\theta)$. Note that the probability amplitude that particle 2 is in the detector at angle θ is $f(\pi-\theta)$, because if particle 2 is at angle θ, particle 1 will be at angle $\pi-\theta$ (in the center-of-mass frame).

Now we would like to know the probability that some particle lands in the detector. How do we combine the two amplitudes? The double-slit experiment corresponded to a *single* particle interfering with itself. Here we have two separate particles. Can they interfere? Here's an important rule:

> You only add amplitudes if the two final states are *indistinguishable* from each other.
> If the states are distinguishable, then you add probabilities.

In the double-slit experiment (without the light bulb), at the backstop you don't know which slit the electron went through, so the two possibilities are indistinguishable, and you add the amplitudes.

Say that in the scattering experiment the particles are distinguishable (i.e. one of them carbon atom, the other a neutron). This means the two possibilities are distinguishable. (It doesn't matter whether or not you actually do distinguish them – the point is that you *could* tell them apart if you wanted to.) The probability that one of the two particles lands in the detector is then the sum of the individual squares:

$$P(\text{some particle at } \theta) = |f(\theta)|^2 + |f(\pi - \theta)|^2$$

There is no interference between the two possiblities. (The interference we discussed in crystal diffraction was the neutron interfering with itself, not with the graphite it scattered from.)

So that seems a fairly plausible generalization of the single-particle situation. There's no interference between different kinds of particles. But now let's consider the same experiment, except that the particles are the same as each other. Let's say that they are two α-particles (these are helium nuclei, they were seen as products of radioactive decay before and named α-particles before it was realized what they were). Again, $f(\theta)$ is the probability that particle 1 goes to angle θ. Here, however, one cannot tell the state $\langle\theta, \pi - \theta|$ from the state $\langle\pi - \theta, \theta|$. This is what it means to say that particles are *identical*. An electron is an electron is an electron: there is no way of telling one electron from another.

Because the two final states are identical, we take a linear combination of the *amplitudes*. For alpha particles, it turns out we combine the two amplitudes with a relative + sign. This means that the probability that we see one of the two alpha particles in the detector is

$$P(\text{some alpha particle at } \theta) = |f(\theta) + f(\pi - \theta)|^2$$

This is a totally different answer than the one for distinguishable particles, even if $f(\theta)$ is the same. For example, say the angle θ is $\pi/2$ (the particles scatter at a right angle to the original). Then here we have $P_+ = 4|f(\pi/2)|^2$. In the distinguishable case, we would have $P_{\text{distinguishable}} = 2|f(\pi/2)|^2$. Thus alpha particles interfere constructively with each other for scattering at an angle $\theta = \pi/2$.

That sounds reasonable, given our double-slit experiments. But now repeat the experiment with electrons. Then you also need to take a linear combination of the amplitudes, but here with a relative minus sign. Namely, you find

$$P(\text{some electron at } \theta) = |f(\theta) - f(\pi - \theta)|^2$$

Notice that if $\theta = \pi/2$ you get zero! Thus electrons interfere *destructively* when they scatter at an angle of $\theta = \pi/2$. In general, when you have two amplitudes related by the exchange of electrons, you *subtract* the two. This doesn't contradict the constructive/destructive interference we saw in the double-slit experiment – that was the electron interfering with itself!