Modern Physics notes – Spring 2007
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Lecture 13

- Emission and absorption of photons
- Bose-Einstein distribution
- Superconductivity and Bose-Einstein condensation

- Feynman, 4.3-4.5
- Fowler, "Blackbody radiation"

Bose-Einstein distribution

A blackbody is a box which contains photons and atoms, so that none can get out. The atoms are in *thermal equilibrium* with the photons. What this means is that we can have transitions which create and annihilate photons, but that the rate at which the photons are being created is the same as the rate at which they're being annihilated. Let's consider just two levels of the atom, with energy difference ΔE. We'll call these the ground state and the excited state. The photon emitted when the atom goes from the excited state to the ground state has frequency $\nu = \Delta E/h$. Likewise, if an atom in the ground state absorbs a photon of frequency ν, it jumps to the excited states.

Statistical mechanics is the study of the large number of particles. One doesn't study each one individually, of course, but can derive things about their aggregate properties from a few basic assumptions. For example, all the laws of thermodynamics you learned about in chemistry or other physics classes can be derived from statistical mechanics. A central issue is how to define temperature. The answer is in comparing the relative numbers of different particles which are in thermodynamic equilibrium. If in thermodynamic equilibrium we have N_g particles in the ground state, and N_e in the excited state, then the temperature T is defined as

$$\frac{N_e}{N_g} = e^{-\Delta E/k_B T} = e^{-\hbar\omega/k_B T}$$

where k is a fundamental constant called Boltzmann's constant; $k_B = 1.38 \times 10^{-23} J/K$. ($k_B$ is simply related to the ideal gas constant: the ideal gas law $PV = nRT$ written in terms of Boltzmann's constant is $PV = Nk_BT$, where N is the total number of particles and n the number of moles. Thus $k_B = R/N_a$, where N_a is Avogadro's number 6.02×10^{23}.)

In thermal equilibrium, photons are being created and annihilated constantly. One would expect that the higher the temperature, the more photons one will have. This is because the atoms have more energy and so collide with each other more often, allowing more energy to be transferred around in the form of photons. The definition of temperature above allows us is very useful for blackbodies – it allows us to find how the average number of photons in the box $\overline{n}$ depends on temperature. So let's use this with our enhancement calculation. Let $\overline{n}$ be the average number of photons with frequency of ω (so it's a function of ω). Then our enhancement result says that the absorption rate from this state is

$$N_g \overline{n} |a|^2$$

It's multiplied by N_g because you can have a transition upward in any atom which is in the ground state. The emission rate into this state is

$$N_e(\overline{n}+1)|a|^2$$

The definition of thermal equilibrium is that these two are equal. Combining these with the relation between N_g and N_e gives

$$\frac{\overline{n}}{\overline{n}+1} = e^{-h\nu/(k_B T)}$$

Solving for $\overline{n}$ gives

$$\overline{n} = \frac{1}{e^{h\nu/k_B T} - 1}$$

This is an amazing result. From these simple laws of quantum mechanics and statistical mechanics, we know the average number of photons of a frequency ν in the box. Particles which obey this formula are said to have "Bose-Einstein statistics", and are called of course "bosons".

Superconductivity and Bose-Einstein condensation

One remarkable thing about bosons is that it is possible to get a *macroscopic* number of them in the *same* quantum state. A laser is one example of this. Here's another way: just cool a gas of bosons enough, and our results above show that most or all of the particles will fall in to ground state of the system.

We derived the formula for $\overline{n}$ for a specific system, but in fact it holds for any bosons in thermal equilibrium in any system. We have in general

$$\overline{n}(E,T) \propto \frac{1}{e^{E/k_B T} - 1}$$

as the average number of bosons $\overline{n}$ of energy E at temperature T. Notice that as the temperature $T \to 0$ (absolute zero), this goes to zero unless $E = 0$. Thus at low enough temperature, all the particles of the system will be in the $E = 0$ ground state. This is called *Bose-Einstein condensation.*

Sounds easy, but it was first done only about 10 years ago (by a guy I went to grad school with!) One needs to cool the atoms down to a mind-bogglingly small temperature, which is done by many clever tricks involving lasers. Now it's done downstairs in this building!

Although BEC was only seen by this method recently, it's been seen in fermionic systems for nearly a century! It's called *superconductivity*, which happens in some materials at low enough temperatures (but far higher than the BEC I mentioned above). Electrons are fermions, and moreover, they repel each other because of the Coulomb interaction. In these materials, however, at low enough temperatures they develop an attractive interaction because of their interacions with the lattice of atoms. The electrons then can pair up into what are called "Cooper pairs". A pair of electrons makes a boson, so it can then occupy the ground state with other pairs. A remarkable property of this state is that current is carried without resistance. Not just a small resistance, but literally zero.

This happens because they are bosons – the Cooper pairs are all in the *same* state. There is an non-zero energy difference (a *gap*) between this state and the lowest-lying excited state. If the temperature is low enough, there is not enough energy to break up the pairs. As long as starting a small current also does not heat up the sample enough to excite it and break up the pairs. People have started a current in a superconducting ring, removed the battery, and watched it run for years. All you need to do is keep the material cold enough.

A related phenomenon Feynman mentions is superfluidity. This seems to occur only in helium. Usually, if you cool something enough, it turns into a solid. But when you cool off helium, it forms a liquid but never a solid. You can then under the right conditions get a macroscopic number of atoms in the ground state, and end up with a superfluid. Helium-3 is a fermion, and so it needs to form pairs to become a superfluid (which it does).

But even BEC is so 10-years ago. In the last few years, people have succeeded in cooling down fermionic atoms to the same mind-bogglingly small temperatures. Then one can hopefully study superconductivity in these systems, where the interactions are small, and can be controlled much better than in a typical material.