Modern Physics notes – Spring 2007

Paul Fendley fendley@virginia.edu

Lecture 2

- Waves
- Diffraction and Interference
- Bullets
- Electron Interference

- Feynman 1.2-1.5
- Fowler, the beginning of "Wave Equations for Photons and Electrons" and "Probabilities, Amplitudes and Probability Amplitudes"

Waves

We just saw in class that when light is shined through small enough gratings, it displays what are called interference patterns.

You've probably already heard the explanation for this: light is a wave. Everybody's seen waves before, in life as well as in physics class. Unfortunately, there was a very large water wave causing colossal destruction a year ago. Waves you're familiar with are some sort of disturbance (in water, in air, in electromagnetic field) which propagates. It repeats itself after a certain amount of time (the period), and after a certain distance in space in the direction of propagation, the wavelength. The wave velocity is related to the wavelength λ and the period T by

$$v = \frac{\lambda}{T}.$$

The inverse period is the frequency: $\nu = 1/T$, and is measured in cycles per second (known in some contexts as Hertz, but best just to think of as inverse seconds). For tsunamis in the Pacific, the wavelength can be as long as 150 miles, and the period can be around 20 minutes, meaning they travel as fast as 450 mph.

Let's focus on electromagnetic waves, which are not as depressing to think about. As I hope you'll remember, one very interesting consequence of Maxwell's equations is that they have wave solutions. (See Fowler's lecture notes for a derivation of this fact.) There are solutions of the equations which propagate in space where there are no charges. In particular, instead of depending on time and space independently, they depend on the combination

$$x - ct$$

where c is the speed of light. Since wave solutions are periodic in space and time, this means that the electric field can be expanded as a series where the terms have the space and time dependence as e.g.

$$\cos\left[\frac{2\pi}{\lambda}(x-ct)\right] \qquad \text{or} \qquad \sin\left[\frac{2\pi}{\lambda}(x-ct)\right]$$

Note that both of these are invariant under shifts $x \to x+\lambda$. and $t \to t+T$, where as above $T = c/\lambda$.

An important property of waves is their *intensity.* For visible light, this is the "brightness". A precise way of defining the intensity for electromagnetism is as the energy density. Since waves cannot have negative energy, the intensity must be positive (and of course real). For an electromagnetic wave, the intensity is

$$\propto |\vec{E}|^2 + |\vec{B}|^2$$

If you use MKS units, there are some relative constants, but these aren't important for the qualitative arguments I'm giving. By $|\vec{E}|^2$, I mean $\vec{E}^* \cdot \vec{E}$; this is merely the magnitude of the vector squared. It makes sense that the intensity involves only the magnitude: electric field is a vector, so of course in some basis it will be negative. Taking the magnitude is one way of always getting a positive number (and something independent of direction – energy is not a vector).

Interference and diffraction

One major consequence of wave-like behavior is that it allows one to simply understand interference and diffraction. One important characteristic of Maxwell's equations and of quantum mechanics is that the equations are *linear.* This means that (in the absence of sources), the sum of any two solutions is a solution. Thus if we have two sources for waves, the total wave is just the sum of the two. This is usually illustrated by the double-slit experiment: the total electric field is just the electric field found by adding together the two electric fields found by treating the two slits as independent sources:

$$\vec{E}_{12} = \vec{E}_1 + \vec{E}_2.$$

The intensity I_{12} then merely depends on $|\vec{E}_{12}|^2$. This sounds obvious, but note that

$$I_{12} \neq I_1 + I_2$$

The electric fields add, but the intensities don't! Indeed, if $\vec{E}_1 = -\vec{E}_2$ and $\vec{B}_1 = -\vec{B}_2$, we have $I_{12} = 0$, even though both I_1 and I_2 are positive and non-zero. This is not just a mathematical curiosity, because intensities are what our eye measures.

The locations of the light and dark spots depends on the wavelength of the light and on how far a given point is from the two sources. This can be seen easily by drawing pictures. It is easiest to look at the situation where both waves have the same wavelength. If the two waves have the same starting point (zero phase difference), they constructively interfere. If the two waves have phase difference π, they destructively interfere, canceling entirely if they have the same amplitude.

So now consider a point on the backstop. It is a distance d_1 from slit 1, and a distance d_2 from slit 2. First consider constructive interference, so that at this point on the backstop, the two waves line up. For this to happen, the difference in distances $d_2 - d_1$ must be an integer n number of wavelengths:

$$d_2 - d_1 = n\lambda \qquad \text{constructive}$$

For destructive interference, the two waves are maximally off:

$$d_2 - d_1 = \left(n + \frac{1}{2}\right)\lambda \qquad \text{destructive}$$

This explanation of interference phenomena were what finally convinced scientists in the nineteenth century that light was a wave and not a particle.

Bullets

To see how different waves are from particles, consider the same double slit experiment done with bullets from an erratic machine gun instead of light. The gun needs to be erratic so we don't know whether a given bullet will go through slit 1 or slit 2 (we can forget bullets that don't go through – our detectors are on the other side).

The analog of the intensity here is the probability that a given bullet will bounce off the wall, through a slit, and hit a given spot on the backstop. Then it's pretty obvious that if P_1 is the probability for when only slit 1 is open, P_2 the same for slit 2, and P_{12} the pattern when both slits are open, we have

$$P_{12} = P_1 + P_2.$$

Thus there is no interference, constructive or destructive. It is like $\delta = \pi/2$ in the above example.

Electron diffraction

Now we come to electrons. In your high-school chemistry or physics class, you probably learned that they are very small particles which "orbit" the nucleus in an atom. In more advanced classes, you may have learned that they are the particles which carry electric charge in a circuit. You may know that a picture tube works by firing a beam of electrons (by an "electron gun") at chemicals called phosphors, which glow different colors when they absorb an electron. Electrons were only discovered at the end of the nineteenth century. One of the shocking things was that their mass was exceptionally small when compared to protons (about a thousandth of it). This small mass is one of the reasons the following effect can be seen.

So let's do the same double-slit experiment with an electron gun instead of light or with bullets. Since elect-ons are particles, not waves, we have to run the experiment like we did for bullets: we have to send in a bunch of electrons, and then find the probability P_1 that a given electron lands in a certain place on the backstop. If we do this, however, we find that electrons do not seem to behave like bullets. We have

$$P_{12} \neq P_1 + P_2.$$

There is interference!

This experiment doesn't disprove the fact that electrons are particles. Indeed, the detector still registers electrons individually. It never sees half an electron. The wave-like behavior comes when we measure the probabilities. The probabilities behave in the same way as probabilities for waves. In the absence of slits, we have functions ϕ_1 and ϕ_2 such that

$$P_1 = |\phi_1|^2 \qquad\qquad P_2 = |\phi_2|^2$$

so that

$$P_{12} = |\phi_1 + \phi_2|^2$$

even though $P_{12} \neq P_1 + P_2$. So ϕ_1 looks just like the electric field for a wave. But what sort of wave is an electron?