Modern Physics notes – Spring 2007
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Lecture 23

- Energy splitting
- The ammonia molecule in an electric field

- Feynman, 9.2-9.5

Energy splitting

We've found the stationary states in the presence of tunneling. One important characteristic is that the energy levels "split". There are two energy levels, one with energy $E_0 + A$, and one with energy $E_0 - A$. Note that this is just like the spin-1/2 atom, where the two stationary states $|+Z\rangle$ and $|-Z\rangle$ have different energies due to the magnetic field in the z direction.

Calculating A is very difficult; it really can only be done numerically. But it can be measured experimentally by observing the frequency of the radiation emitted when the maser makes a transition between these two levels. One finds that it's about $24000\,MHz$, so

$$E_I - E_{II} = 2A = h\nu$$

Thus A is about $10^{-4}\,eV$, so this splitting is much smaller than that of the energy levels in an atom. $10^{-4}\,eV$ is about $1\,cm$; these are microwaves.

The ammonia molecule in a constant electric field

To make a maser, you need a way to control transitions between the two states $|I\rangle$ and $|II\rangle$. This is done by introducing an electric field. The effect is very similar to putting a spin in an magnetic field. The ammonia molecule has a net zero charge, but because of the way they bond, the electrons tend to lie closer to the nitrogen than the hydrogen. Thus the region around the hydrogen atom has a slight net positive charge. This makes an electric dipole, whose direction depends on which side the nitrogen is on. When you put an dipole $\vec{\mu}_{\mathcal{E}}$ in an electric field $\vec{\mathcal{E}}$, the energy is $U = -\vec{\mu}_{\mathcal{E}} \cdot \vec{E}$. To simplify matters a little, assume that $\vec{\mu}_{\mathcal{E}}$ is aligned or antialigned with $\vec{\mathcal{E}}$, so that $U = \pm\mu_E E$.

It seems like we can now just take over our results for spins in a magnetic field. There's a catch, though. Before we introduced the electric field, the states $|1\rangle$ and $|2\rangle$ were like the states $|+X\rangle$ and $|-X\rangle$; including the tunneling is like placing a magnetic field in the z direction. They aren't the stationary states: the combinations $(|1\rangle \pm |2\rangle)/\sqrt{2}$ are. But now we're putting on an electric field which splits the energies of the $|1\rangle$ and $|2\rangle$ states. In the spin-1/2 problem, this would be like adding a magnetic field in the x direction, along with the one already present in the x direction. This means the stationary states in the presence of both tunneling and the electric field are neither $|I\rangle$ and $|II\rangle$, nor are they $|1\rangle$ and $|2\rangle$.

What we need to do is solve the differential equations for the stationary states in the presence of the electric field. We won't go through the whole thing, but I'll show you how to extract the energies of the stationary states. Like last time, first consider what happens in the absence of tunneling. Then the states $|1\rangle$ and $|2\rangle$ are stationary states, with energies $E_0 - \mu_{\mathcal{E}}\mathcal{E}$ and $E_0 + \mu_{\mathcal{E}}\mathcal{E}$. The symmetry between states $|1\rangle$ and $|2\rangle$ is broken due to the electic field. Now include the tunneling amount A. We make the reasonable assumption that the amount of tunneling doesn't depend on $\mathcal{E}$. The differential equations are then:

$$
\begin{aligned}
i\hbar\frac{\partial C_1}{\partial t} &= (E_0 + \mu_{\mathcal{E}}\mathcal{E})\, C_1 - AC_2 \\
i\hbar\frac{\partial C_2}{\partial t} &= (E_0 - \mu_{\mathcal{E}}\mathcal{E})\, C_2 - AC_1
\end{aligned}
$$

There are some general techniques for solving equations like this. The trick to get the energy is to recognize that there will still be two stationary states, but they no longer will be $|I\rangle$ and $|II\rangle$. Let's label the energies of the stationary states in the presence of the electric field as $E_{\mathcal{E},I}$ and $E_{\mathcal{E},II}$. We know that for a stationary state of energy E, both C_1 and C_2 depend on energy and time as e^{-iEt}. For example, in the stationary state with energy $E_{\mathcal{E},I}$

$$
C_1 = a_1 e^{-i\mathcal{E}_{\mathcal{E},\mathcal{I}}t} \qquad C_2 = a_2 e^{-i\mathcal{E}_{\mathcal{E},\mathcal{I}}t}
$$

with a_1 and a_2 independent of time. So now you can just plug into the differential equations, and solve for E. You get a quadratic equation for E: the two solutions are the energies of the two stationary states. In the case where the electric field $\mathcal{E}$ is independent of time, you get

$$
\begin{aligned}
E_{\mathcal{E},I} &= E_0 + \sqrt{\mu_{\mathcal{E}}^2\mathcal{E}^2 + A^2} \\
E_{\mathcal{E},II} &= E_0 - \sqrt{\mu_{\mathcal{E}}^2\mathcal{E}^2 + A^2}
\end{aligned}
$$

It's always good to check your answer when deriving complicated equations. So check the cases $\mathcal{E} = 0$ and $A = 0$ to make sure you get the same answers we had before.

So we've now found what happens to the ammonia molecule in an electric field. We can thus split the beam by having regions of varying electric field. The states want to go to where they have the least energy. The staionary state with energy $E_{\mathcal{E},I}$ will go to regions with less electric field. The state with energy $E_{\mathcal{E},II}$ will go to region of greater electric field, just like the spins in a magnetic field.