Modern Physics notes – Spring 2007
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Lecture 24

- The maser/laser
- The quantum computer

- Feynman, 9.2-9.5
- on the RSA algorithm: http://world.std.com/~franl/crypto/rsa-guts.html

The ammonia molecule in an oscillating electric field

We've found what happens to the ammonia molecule in a electric field constant in time. We can thus split the beam by having regions of varying electric field. The states want to go to where they have the least energy. The state $|I\rangle$ will go to regions with less electric field. The state $|II\rangle$ will go to region of greater electric field, just like the spins in a magnetic field.

To make a maser, you need to put the molecules in an electric field which depends on time as $\mathcal{E} = 2\mathcal{E}_0 \cos(\omega t)$. Solving this is a little more complicated. The experimentally-relevant situation is where the energy in the electric field $\mu_{\mathcal{E}}\mathcal{E}$ is much smaller than the amount of tunneling A. In this limit the energies E_I and E_{II} of the stationary states are close to their values $E \pm A$ in the absence of an electrical field.

What one finds in an oscillating electric field is an example of something you've seen (probably several times) in mechanics. You find a *resonance* phenomena. The point is that because $\mu_{\mathcal{E}}\mathcal{E} \ll A$, in general, it can't force the atom to make the transitions between stationary states I and II (the tunneling barrier is too high). The only way you can cause it to make transitions is if you get a resonance. The splitting between the levels is $2A$. This corresponds to a frequency

$$\omega_0 = \frac{2A}{\hbar}$$

If you make the frequency of the electric field ω the same as the "resonant" frequency ω_0, you get a resonance. This is how you make the wine glass break by tuning the frequency of sound: the sound only gives a little push to the glass, but if its frequency is just right, the oscillations get bigger and bigger. When you push someone on a swing, you don't need to push it particularly hard, as long as you time your pushes well: the frequency of the pushes needs to be the same frequency the swing is moving at. So if the frequency of the electric field ω is the same as $\omega_0 = 2A/\hbar$, you can cause transitions.

You can look through Feynman to see the details. In the presence of a small electric field, the states $|I\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$ and $|II\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$ are no longer stationary states. You find that $\omega = \omega_0$ (i.e. resonance), the electric field $\mathcal{E} = 2\mathcal{E}_0 \cos(\omega t)$ causes transitions between the two states. Assuming it starts in state $|I\rangle$ at time $t = 0$, the formula describing the probability it's in state $|I\rangle$ at some later time t is

$$P_I = \cos^2\left(\frac{\mu_\mathcal{E}\mathcal{E}_0}{\hbar}t\right)$$

and (of course)

$$P_{II} = \sin^2\left(\frac{\mu_\mathcal{E}\mathcal{E}_0}{\hbar}t\right)$$

Not surprisingly, you find that if $\omega \neq \omega_0$, you don't get transitions: the probability does not oscillate, but rather the system stays in stationary states with probability essentially 1 (the tunneling is exponentially supressed). In particular, Feynman shows that you only get an appreciable transition probability when $(f - f_0)/f_0$ is less than 10^7. This is (one of the reasons) you get such accurate time with atomic clocks: nothing happens unless the frequency of the radiation involved takes on an extremely precise value.

Masers and lasers

So here's the trick to make a maser work. In the absence of an electric field, there are two stationary states $|I\rangle$ and $|II\rangle$, with energy $E + A$ and $E - A$ respectively. Thus for A positive (the experimental situation), $|I\rangle$ is the excited state and $|II\rangle$ the ground state. By putting a beam of ammonia molecules in a spatially-varying field you can separate the two states. For the maser we want a beam in the *higher* energy state $|I\rangle$.

Having obtained the beam of $|I\rangle$, we send it in to a cavity. Here's the fiendishly clever part. In the cavity you have an electric field oscillating at the frequency $\omega = \omega_0$. This then causes the state $|I\rangle$ to oscillate into $|II\rangle$. By energy conservation, this then *releases* radiation, because $|I\rangle$ has higher energy than $|II\rangle$. Let's check the frequency of this radiation. The energy difference $(E_0 + A) - (E_0 - A) = 2A$, so the frequency of this radiation is $2A/\hbar$. But this is just ω_0 !! So this puts radiation into the cavity at exactly the right frequency! So sending the beam through the cavity puts more and more energy into the cavity in the form of radiation at exactly the right frequency ω_0. This process can only emit energy at this frequency. Moreover, if atoms in the wrong state get sent in (or some other process happens), this won't have an appreciable effect: the resonance is so sharply peaked around ω_0.

This is called *stimulated emission* and this gives masers and lasers their name. It is a way of getting a macroscopic (i.e. enough so that laser light is visible) number of photons in the same quantum state. If you set up your mirrors right, then they're all going in the same direction, and if you poke a little hole in the wall (or have a partially silvered mirror), you get a laser beam! The beam is *coherent*. This means that all the photons are indistinguishable: they are moving in the same direction with the same momentum (because they all have the same frequency and wavelength, they all have the same momentum: $p = h/\lambda$.)

Real lasers usually use a system with more energy levels. The reason is that you don't want the photons being absorbed and re-exciting the system back from $|II\rangle$ to $|I\rangle$. Thus to make a laser find a system where (a) the state $|I\rangle$ is a long-lived excited state, and (b) $|II\rangle$ is a *short*-lived excited state of *lower* energy. Thus only when stimulated, does the state $|I\rangle$ go to $|II\rangle$ and release a photon. Then the state $|II\rangle$ quickly decays to a state of even lower energy, the ground state $|0\rangle$. Then the photons can't excite $|0\rangle$ to anything; you excite it back to $|I\rangle$ by other means (just having it a room temperature means that collisions between the atoms will excite them). There are of course technical complications as well in actually getting a maser or laser to work. But the important point to remember is that the key underlying physics is still that of the two-state system.

Two-state systems and quantum computers

A spin-1/2 particle at rest is just one example of a *two-state system*. The ammonia molecule we just discussed is another. The nitrogen atom can be on either side of the three hydrogen atoms. Another is a laser, where the main physics comes from a photon moving in between two energy levels. Of course, these systems are much more complicated than just these two states, but the crucial physics we're looking for (maser, laser), come from transitions between just these two states. This is why ignore the rest of the system and just focus on these two states.

One of the many amazing things about quantum mechanics is all of the non-trivial physics which can come out of such a system. The reason is that in quantum mechanics, when we say a "two-state system", we don't really mean that there are only two possibilities for the system: there are an infinite number of states. Why we call it a two-state system is that there are just two *basis* states. Let's call these two basis states $|1\rangle$ and $|2\rangle$ As we've seen above, this means that the system can be in the state

$$|S\rangle = A|1\rangle + B|2\rangle$$

for any complex numbers A and B. (This sounds like four arbitrary real parameters, but in fact there are only two that matter: the overall normalization $\langle S|S\rangle = 1$ fixes $|A|^2 + |B|^2 = 1$, and the overall phase doesn't matter, really, since it doesn't affect the probability). We thus have an infinite number of possible states governed by these two numbers. However, if we *measure* the state of the system: we just get either $|1\rangle$ or $|2\rangle$. This is why it's fair to call it a two-state system.

A bit in a computer is also a two-state system, usually denoted by zero or one. This is a classical two-state system, so it really just has two states. A "quantum computer" uses a *quantum* two-state system for each bit. Each of these is called a "qubit". If one could manipluate these systems well enough (which experimentally is not possible yet), then one could store information in qubits instead of ordinary old classical bits. Because of the peculiar property of quantum mechanics that you can take linear combinations of states, this enables one to do certain kinds of computations (e.g. a reverse phone book) much faster than you can on a conventional computer. Feynman, in the last decade of his life, was a big proponent of quantum computation – you can buy the Feynman lectures on quantum computation. But the major development that got people excited about quantum computers came from a mathematician named Peter Shor, who developed an algorithm which allows a quantum computer to factorize a number into its prime factors in time increasing as a polynomial time.

This sounds like a mathematical curiousity, but if one could build a quantum computer which could factorize large numbers, then this would pretty much wreck our web-based economy. There's an interesting branch of computer science and mathematics where they prove theorems about how long it takes to solve problems. One problem of great importance is how long it takes to factor a number into its prime components. It is believed (although not yet proven) that as the number gets higher, the time to factor it using a conventional computer increases exponentially. The reason it is important is that codes you use all the time (e.g. when using a secure connection over the web) rely on this – it's fairly easy computationally to find 100-digit prime numbers, and easy to multiply two of them together to get a 200-digit number. It's basically impossible to factorize this on a conventional computer, and the encryption uses this fact. You can find a simple explanation of the coding algorithm (called the RSA algorithm) at the above web address. This form of encryption is particularly useful because it is has a *public key* – you can tell anyone how to encode a message, but only you know how to decode. The public-key encryption is a great thing – it allows you encrypt your credit card number to send over the web without having to arrange a key in advance (any address beginning with `https://` involves encryption).

A quantum computer would render it impossible to do this safely anymore. Luckily for capitalism, building one seems to be a long way off. It's very hard to make a system which is both quantum mechanical and where you can manipulate individual atoms. So far, they've built a quantum computer which factorized the number 15 (it did confirm that this is indeed 5×3). But given the stakes, many people are trying.