Modern Physics notes – Spring 2007

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Lecture 27

- Angular momentum and positronium decay
- The EPR "paradox"

- Feynman, 18.3, 11.4
- Blanton, http://math.ucr.edu/home/baez/physics/Quantum/bells_inequality.html
- Quantum teleportation: http://www.research.ibm.com/quantuminfo/teleportation/

Angular momentum and positronium decay

Symmetry is of enormous importance in physics. When a system has symmetry, it leads to conservation laws: symmetry under time translation (i.e. the laws of physics are the same now as they were in 400 B.C.) results in energy conservation. Symmetry under spatial translation results in momentum conservation. Symmetry under rotations results in angular momentum conservation.

To understand this in depth is beyond this course. But one can see some very simple consequences of how symmetry affects quantum mechanics (and hence the world!). I'll explain how conservation of angular momentum helps us understand the decay of a particle called "positronium". Understanding this will lead to a seeming paradox, but whose resolution shows that quantum mechanics is correct and consistent, but very weird.

Positronium is a bound state of an electron and a positron. A positron is an anti-electron: it has the electron mass, the same spin-1/2, but the *opposite* charge of an electron. (It therefore has positive charge, hence the name.) We haven't talked about anti-particles much, and won't have to say much more. Suffice to say that Dirac showed that in order to have a consistent theory of both quantum mechanics and special relativity, you must have antiparticles. Subsequently, the positron, anti-proton, etc., were observed. Since the photon is zero charge, it can be (and is) its own antiparticle. Thus the anti-photon is the same thing as a photon.

We don't see antiparticles very often, although they're easy to make in high-speed particle collisions (the kind that happen in accelerators like Fermilab and JLab). The reason we don't see them is that a particle and an anti-particle can *annihilate* each other. A particle and antiparticle together have no net charge, so it does not violate charge conservation for them to annihilate into photons (it needs to be more than one photon to conserve energy and momentum). When we do special relativity, we'll see how one can understand energy conservation in such decays (advance preview: $E = mc^2$).

Positronium is sort of like hydrogen, with the positron playing the role of the proton. But as opposed to hydrogen, the positron is not stable: after about 10^{-10} seconds, the positron and electron annihilate into photons. Thus we say that positronium decays, like a radioactive nucleus.

But now here's the interesting part. Let's look at the spin of positronium. The electron and positron each can have $S_z = \hbar/2$ or $S_z = -\hbar/2$. There are therefore four possible states for the spins of an electron and a positron:

$$|\uparrow\uparrow\rangle,\ |\uparrow\downarrow\rangle,\ |\downarrow\uparrow\rangle,\ |\downarrow\downarrow\rangle$$

Remember that although we can not in general know all the components of the spin at one time, we can know both S_z and $\vec{S}\cdot\vec{S}$. To get the total S_z, you just add the S_z of each. The four possible spin states of positronium therefore have $S_z = \hbar, 0, 0$, and $-\hbar$ respectively.

Finding $\vec{S}\cdot\vec{S}$ for positronium is trickier. Remember that a particle is of spin-s when $\vec{S}\cdot\vec{S} = s(s+1)\hbar^2$. For example, for a spin-1/2 particle, $\vec{S}\cdot\vec{S} = 3\hbar^2/4$: this is easy to check by remembering that always $S_x = \pm\hbar/2$, $S_y = \pm\hbar/2$ and $S_z = \pm\hbar/2$. This means that always $(S_x)^2 = \hbar^2/4$, $(S_y)^2 = \hbar^2/4$ and $(S_z)^2 = \hbar^2/4$, so that $\vec{S}\cdot\vec{S} = 3\hbar^2/4$ as claimed. For general spin s, S_z can have the quantized values

$$S_z = -s\hbar, (-s+1)\hbar, \ldots, (s-1)\hbar, s\hbar$$

This means there are $(2s+1)$ possible spin states of a spin-s particle.

There are four possible spin states of positronium, so you might guess that positronium is of spin 3/2. However, this can't be: the four possible S_z values of a spin 3/2 particle are $-3\hbar/2,\ -\hbar/2,\ \hbar/2,\ 3\hbar/2$, while the four S_z values of positronium are $S_z = -\hbar,\ 0,\ 0,\ \hbar$. Thus what happens is that there are two possible spin values for positronium. A positronium particle can either have spin 1 (and so $S_z = \hbar,\ 0,\ -\hbar$) or spin 0 (and so $S_z = 0$). This is possible because positronium is a bound state of fundamental particles.

This explanation leaves one question still open. Of the two states with $S_z = 0$, which has $\vec{S}\cdot\vec{S} = 0$, and which has $\vec{S}\cdot\vec{S} = 2\hbar^2$? On your homework, you checked that the state

$$\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle \ - \ \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

is the one with $\vec{S}\cdot\vec{S} = 0$.

We need to know one more thing about spin before understanding positronium decay. Photons have spin 1. Let the z axis be the direction the photons is moving (remember light is never at rest, so there always is a well-defined z axis for a photon.) When $S_z = \hbar$ for a photon, we say the photon is *right-circularly polarized*. Classically, this means that the electric field vector is rotating clockwise, hence the name. When $S_z = -\hbar$, it is *left-circularly polarized*. You may recall that light cannot be longitudinally-polarized: this means that the electric and magnetic fields always point perpendicular to the direction of motion. The quantum-mechanical version of this statement is that S_z cannot be zero for a photon, it can only be $S_z = \pm\hbar$.

Finally, we can understand the decay of positronium. Let's consider the possible decay: positronium with $\vec{S} \cdot \vec{S} = 2\hbar^2$ (i.e. spin 1) at rest decaying into two photons. By momentum conservation, the two photons head off in opposite directions. Because angular momentum is conserved in the decay (remember that spin is just intrinsic angular momentum!), the two photons after the decay must have the same S_z as the positronium did before the decay. Two photons heading in opposite directions have four possible spin states

$$|RR\rangle,\ |RL\rangle,\ |LR\rangle,\ |LL\rangle$$

(there would be nine if longitudinally-polarized $S_z = 0$ photons existed). These four states have possible $S_z = 0,\ 2\hbar,\ -2\hbar$ and 0. (Note that $S_z = 0$ for $|RR\rangle$: the z axis for a given photon is its direction of motion, and the two photons are moving in opposite directions.) Thus positronium with $S_z = \hbar$ or $S_z = -\hbar$ cannot decay into two photons. In fact, a state with $\vec{S} \cdot \vec{S} = 2\hbar^2$ cannot ever decay into two photons, even if the overall $S_z = 0$. The reason comes from the fact that photons are bosons. When you rotate the system by 180 degrees, the two photons just change places. Since they are identical (both have $S_z = 0$) bosons, the probability amplitude must be the sum of the two possibilities. However, you can show (using some rules slightly beyond the scope of this course, but they're in Feynman, chapter 17) that if you rotate a state with $\vec{S} \cdot \vec{S} = 2\hbar^2$ and $S_z = 0$ by 180 degrees, you pick up a minus sign. This means that the two amplitudes have opposite sign. When you add the two, you get zero!

Therefore a spin-1 positronium particle cannot decay into two photons. This is an example of how powerful symmetry can be in physics. It means that it can only decay into 3 photons, and it turns out that the amplitude for this to happen is much smaller than that for two-photon decay (to compute this requires quantum field theory, a topic that unfortunately even many graduate students in physics don't bother with). Roughly speaking, the reason is that you get a contribution e^2 for every photon involved, so in the first case the decay rate is proportional to e^4, while in the second it's proportional to e^6. Since e^2 isn't dimensionless, one needs to do more work, but the dimensionless combination

$$\frac{e^2}{4\pi\hbar c} \approx \frac{1}{137}$$

Thus spin-1 positronium lives approximately 100 times longer than spin 1/2. When one does the full computation, one finds that the half-life of spin 1 is about 10^{-7} seconds, and spin-0 about 10^{-10} seconds.

Positronium with spin 0 (which requires $S_z = 0$) does decay into two photons. The final state must have all components $S_x = S_y = S_z = 0$. The photons must therefore be in the state

$$\frac{1}{\sqrt{2}}|RR\rangle - \frac{1}{\sqrt{2}}|LL\rangle$$

just like a $\vec{S} \cdot \vec{S}$ state of two spin 1/2 particles $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$.

The EPR "paradox"

As we've seen, the fact that a quantum-mechanical state can be a combination of multiple states has a number of interesting and somewhat counter-intuitive consequences. One interesting one is known as the Einstein-Podolsky-Rosen "paradox". As we'll see, it's not really a paradox: it makes no contradictory predictions, but the prediction it does make is certainly weird.

Let's consider spin-0 positronium decay into two photons. We just showed that this two-photon state must be

$$\frac{1}{\sqrt{2}}|RR\rangle - \frac{1}{\sqrt{2}}|LL\rangle$$

The two photons are traveling in opposite directions at the speed of light. Now let the two photons travel far apart (say a light-year), and measure the spin of one of them. Half the time you'll get R, half the time you'll get L. That doesn't seem so funny – we've seen that many times. But now let's think about the other photon. Note that there is no amplitude for $|RL\rangle$ or for $|LR\rangle$. Thus if we measure that one photon is R, then the other *must* be R as well. If we measure that one is L, then we know that the other one must be L as well.

This is incredibly weird. Say the positronium atom decays halfway between here and Alpha Centauri. This result says that if I do a measurement here, then I affect the measurement at Alpha Centauri at the same time! If I don't do the measurement here, then at Alpha Centauri they might measure either R or L. If I do the measurement here and get R, they must get R. If I get L, they must get L. The same applies for Alpha Centauri. Say the positronium was slightly closer to Alpha Centauri, so the photon gets there first. If they do the measurement and get R, we must get R. If they get L, we must get L.

It gets even weirder if you then look at the linear polarazations. On the HW you'll show how to relate circular polarizations to linear ones. For a single R photon, you would have a 50% chance of measuring x polarization, and 50% of measuring y polarization. For a single L photon, you have the same probabilities (just like in the spin-1/2 case when you change from z to x spins). However, if you work out the state of two photons with $\vec{S} \cdot \vec{S} = 0$, you discover that if you measure x polarization, then Alpha Centauri must measure y polarization, or vice versa. You cannot both measure x under any circumstance. *There is no classical way to get 50% xy, and 50% yx.*

Thus in this sense quantum mechanics seems non-local. A measurement here affects a measurement arbitrarily far away made at the same time. This seems to violate what is called "causality": as we'll see when we do special relativity, nothing can travel faster than the speed of light. This means in particular that the universe is causal: you can only affect something if a light beam can get from one event to another. But what we've shown is that quantum mechanics seems to violate this: what we do here affects the measurement on Alpha Centauri at the same time, and vice versa.

Einstein, Podolsky and Rosen noted this, and then postulated that quantum mechanics must therefore not be complete: there are "hidden variables" which result in the Alpha Centauri measurement being independent of ours. Thus they believed that quantum mechanics is a usually-correct way of describing reality, but that a local theory would replace it eventually. The truth is that this non-locality seems to be experimentally correct! (up to some subtleties mentioned in the web article). A measurement here really will affect the measurement on Alpha Centauri. There is no paradox: reality is just weird.

One can construct other experiments with equally weird results. Say we have a three-particle decay where the particles are 120 degrees apart. As discussed in detail on this homework, there exists a quantum-mechanical state which can only be realized if there is a "spin uncertaintly principle": if you know both S_x and S_y at the same time, this state cannot be realized. Thus there is an *experiment* which can rule out hidden variable theories. In fact, there's a theorem (called Bell's theorem) which says that you must have non-locality in the sense described above in quantum mechanics, and that this has experimental consequences. Unfortunately, it has not yet been possible to do this experiment, but people have done similar experiments. All point to this form of non-locality of quantum mechanics, although as far as I understand there is still a slight possibility quantum mechanics could be wrong and a local, classical theory be correct.

Now one major caution: people have thought that this effect means that you can send information faster than the speed of light. But this is not true: there is no way you can use this non-locality to send a message. When we measure R here we know that Alpha Centauri has R as well. But that can't be used to send a message. We have no way of knowing whether on Alpha Centauri they did the measurement of R and thus forced ours to be R, or we just measured R because there was a 50% chance of doing so. No matter how you construct your experiments, you can't get a message sent going faster than the speed of light. (This didn't stop people from applying for grants to build faster-than-light communication devices for the Army!)

However, one can do very strange things by exploiting EPR-type effects in quantum mechanics. One which got a fair amount of news play a few years back is that it is possible to do "quantum teleportation". One can recreate an exact replica of some quantum state at a different place, but only if one destroys the original quantum state!