Modern Physics notes – Spring 2007
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Lecture 29

- Special Relativity
- Simultaneity (and lack thereof)
- Lorentz transformations

- Born, chapter VI
- Fowler, "Special Relativity", "Time dilation and length contraction"

Special Relativity

To write the two fundamental facts of special relativity, we need to define an *inertial frame*. An inertial frame is one which is not undergoing any acceleration. Thus it is moving at a constant velocity. There are an infinite number of inertial frames, corresponding to all possible velocities. The fundamental facts are

1. The laws of physics are the same in all inertial frames. In other words, there is no preferred frame.

2. The speed of light in a vacuum is measured to be the same value c in any inertial frame, no matter the original source of the light.

These two facts sound fairly innocuous. But we'll go into great detail on showing how this can lead to many counter-intuitive facts.

Simultaneity

In order to describe things precisely, we describes things in terms of "events". An event takes place at a single point in space and a single point in time. Examples of events are an (infinitely small) baseball hitting a board, a flashbulb going off. Note that you just sitting there is not an event. First of all, you're an extended object, so you occupy more than one point in space. Next, you're still there at the end of this sentence. So you exist for than just a single instant in time.

One concept we have to junk with special relativity is absolute simultaneity. Events which take place at the same time in one inertial frame do not take place at the same time in another. If you accept the laws, this is quite easy to see. Let's say you're in the exact middle of a non-moving train car, and send out a light pulse at the same time to the back and to the front. Then you would say that the light lands at the ends of the car at the same time. To put this in our new language, we would say that there are two events, the light hitting the front F, and the light hitting the back B. These two events are simultaneous, as far as you're concerned, because they happen at the same time.

Now let's let the train move at some constant velocity v. You're still in the car, and so you're moving along with it. Thus the train car forms a perfectly good inertial frame, and the laws of physics inside are the same as when you weren't moving. Thus if you repeat the experiment, you'll get the same result. You'll still say that the two events are simultaneous.

But now I'm watching you from the ground as you move by. I will say that the two events are *not* simultaneous. From my point of view, I see the light being emitted at some time, let's call this $t = 0$. One pulse moves to the right at speed c, the other moves to the left at the same speed c. But because the train is moving to the right at speed v, I see it hitting the back of the train first: while the light is in motion, the train moves forward. It is moving toward the left-going light, so it takes shorter for the light to get there. Likewise, the front of the train is moving away from the light pulse, so I say that it takes longer to reach the front. I say the events are not simultaneous!

We can make this clear by drawing a graph in *spacetime*. Let's consider two frames. There's our frame (called S), while the other frame (called S') is moving relative to us at a speed v to the right. Thus think of the situation again as a train car moving by us. (We'll do everything in 1d for now, the generalization is very simple.) It's very useful to draw pictures, which are plots of particles or objects moving in time. Thus label our axes by x and ct. It is convenient to use ct instead of t because then both axes have the same dimensions. It's also conventional to make time on the vertical axis.

The first useful thing to do is figure out how the coordinate axes x' and ct' move in our frame. Here's what this means. On the train, there's a point which they label as $x' = 0$. In S', this point is of course motionless. In our frame, though, it moves with velocity v. By convention, let's choose the time coordinates so that $x' = 0$ is at $x = 0$ at time $t = 0$ and $t' = 0$. In other words, the event $(x, ct) = (0, 0)$ is the same as the event $(x', ct') = (0, 0)$. At later times, the point $x' = 0$ has moved in our frame. It's simple to figure out how, since it's moving with speed v in our frame. Thus the point $x' = 0$ moves to $x = vt$ after a time t (as measured in our frame). This is the ct' axis (the axis, by definition, the point $x' = 0$ at all time). This line has a slope of

c/v, because $\Delta(ct)/\Delta x = c/v$ in our frame. (As a check, notice that as $v \to 0$, the slope becomes infinite: the ct and ct' axes are the same, which of course must be true if the two frames aren't moving with respect to each other.)

The ct' axis was easy. Finding the x' axis is a little harder. The x' is defined as the axis where events in the S' frame are simultaneous. It's easiest to compute this by drawing pictures. We need to find the slope of this line. Go back to the earlier experiment, where we know that the two events are simultaneous in S'. Consider a train car which we measure to be a length L (we'll see later that on the train, they'll measure a different length!). Then we can compute the time t_{back} it takes in our frame to hit the back. We have $L/2 - vt_{back} = ct_{back}$ so that

$$t_{back} = \frac{L}{2(c+v)}$$

Likewise, we have $L/2 + vt_{front} = ct_{front}$ so

$$t_{front} = \frac{L}{2(c-v)}$$

a longer time. The difference $t_{front} - t_{back}$ then gives the time interval between the two events. To compute the slope of the x' axis, we need then to find the distance between these two events. The distance the first beam travels is $x_{back} = x_0 - ct_{back}$, while the distance the second beam travels is $x_{front} = x_0 + ct_{front}$. x_0 is the point at which the beam starts. The slope is

$$\frac{ct_{front} - ct_{back}}{x_{front} - x_{back}} = \frac{ct_{front} - ct_{back}}{ct_{front} + ct_{back}} = \frac{Lv/(c^2 - v^2)}{Lc/(c^2 - v^2)} = v/c$$

The higher the velocity, the steeper the slope. The fastest it can go is $v = c$, where it points at 45 degrees. This is pretty weird. Before Einstein, we would just have the x' the same as the x axis: what's simultaneous in one frame is simultaneous in another.