Modern Physics notes – Spring 2007

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Lecture 3

- Electron Wavelength
- Probability Amplitude
- Which slit?

- Born, IV.4
- Feynman, 1.6-7, 2.1
- Fowler, "Rays and Particles"

The wavelength of an electron

We've seen experimentally that electrons interfere like waves do. It's always then a good idea to push the idea further, and see if the electron interference is quantitatively the same as that for light. In particular, does an electron have a wavelength like light does?

We can test this experimentally by checking the formulas $d_2 - d_1 = n\lambda$ for constructive interference, and $d_2 - d_1 = (n+1/2)\lambda$ for destructive. These formulas work for light waves, and in fact provide an extremely accurate way of measuring lengths. So one can then check the formulas for electrons, by locating the spots of highest and lowest probability of the electrons, and by then measuring the distance of these points from the two slits. One finds all the entire interference pattern obeys the above formulas with a single wavelength. Electrons have a wavelength!

So what does this mean? When confronted with this situation, the best experimental thing to do is to start to vary parameters. As opposed to light beams, electron beams do have something one can vary: the speed of the electrons. The electrons in an electron gun are accelerated simply by placing a voltage between two plates. The kinetic energy of a particle accelerated in a voltage is simply the charge of the particle q times the voltage difference V:

$$KE = qV$$

In MKS units $e = 1.602 \times 10^{-19} C$, and voltage is measured in volts, which are J/C. Since the charge of an electron is so tiny, in these sorts of experiments, it is much more convenient to a different unit for energy, the electron-volt (eV). This is simply the kinetic energy of a single electron accelerated through one volt. Thus to change the speed of the electrons in the electron gun, one merely changes the voltage – this is the knob I was turning last time.

So if one does the same interference, one finds the interference pattern changes when the velocity of the electron changes. The same formulas hold, but different velocities have different wavelengths. One then finds an exceptionally important formula relating the momentum p of a particle to its wavelength:

$$p = \frac{h}{\lambda}$$

The proportionality constant h is called *Planck's constant*, and in MKS units is

$$h = 6.626 \times 10^{-34} J \cdot s$$

You can easily convert this to $eV \cdot s$ if desired.

Now we have a precise definition of the wavelength of a particle. This is often called the de Broglie wavelength, in honor of the physicist who first guessed the formula. Note that it's written in terms of the momentum, not the velocity. There are several reasons. The first is that the formula in this form still applies when the electrons are moving at relativistic speeds, and one no longer has $p = mv$. The second is that the same formula holds for light as well as electrons. In fact, it was derived first for light, and we'll soon see why.

Probability Amplitude

We've seen that electron has a wavelength like a wave. Last time we saw that it has an amplitude like a wave as well: remember that in the double-slit experiment that $P_1 = |\phi_1|^2$ and $P_2 = |\phi_2|^2$, then $P_{12} = |\phi_1 + \phi_2|^2$. This suggests that just like a light wave is described by the electric field, a particle be described by this "wave function" $\phi(x)$. We can't measure this wave function directly, but we can measure $|\phi(x)|^2$. This is the probability that a particle is in a given place x. Thus $\phi(x)$ is also called the "probability amplitude". When you square it, you get the probability.

There is much much more to be said about the probability amplitude: it is one of the fundamental quantities in quantum mechanics. But before doing that, let's explore the double-slit experiment more. This will not only shed more light on electrons, but leads us to understanding the quantum mechanics of light as well.

Which slit does an electron go through?

With electrons, what is interfering with what? If electrons are particles, how can the answer not be just the sum of answers for the two individual slits? If it were just the sum, then we would be well justified in saying that like the bullets, the electron goes through either slit 1 or slit 2. Somehow the two slits are interfering with each other. But how can this be? We could run this experiment where the electron gun waits a while in between firing. We would still get the same results (since we are only measuring a probability): the probabilities don't add. We thus must conclude that there is no way of telling whether the electron goes through slit 1 or slit 2, because if we could, they would be just like bullets. The electron interferes with itself!

This very strange idea can be tested experimentally. Light interacts with electrons, so if we place a light by the slits, we can "see" which slit the electron has gone through. If we do this experiment, we indeed see an electron going through one slit or the other. An electron is indeed a particle like a bullet. How do we reconcile this with interference? We don't! The reason is that when you do this experiment with electrons and the light, one no longer sees the interference. One gets the same results as the for the bullets: the probabilities just add.

So the only explanation for this is that the electrons are very sensitive: turning on the light affects how they behave. This is not shocking: light has energy, and scattering light off of electrons changes the electrons. So maybe we should lower the intensity of the light. But this doesn't solve the problem. It just means that when the light is dim enough, we just start missing electrons. And we find that for the electrons we see, we indeed get $P_{12}^{see} = P_1 + P_2$. For the electrons we miss, we don't know which slit they went through. For these electrons, we have $P^{miss} \neq P_1 + P_2$: they interfere!

So we haven't shed much light on the nature of interference. Is there any way of seeing the electrons without disturbing them? Let's think about light for a second. Because it is a wave, it has a wavelength. If the wavelength is too large, there's no way we can tell which slit the electron went through. So let's try in our experiment increasing the wavelength of the light. For long enough wavelength, we indeed can't tell which slit the electron went through. But then we recover interference! For an even longer wavelength, the light doesn't affect the electron at all: we recover the P_{12} in the absence of light. We can still tell that the electron went through, but we can't tell which slit.

We thus have no logical contradictions: if we know which slit the electron goes through, they don't interfere. The detection will change the system and change the results. If we can't find which slit the electron goes through, then the electrons can interfere. This is our first example of the uncertainty principle, which here says that *any* apparatus sensitive enough to measure which slit the electron goes through will disturb the interference pattern.

More weirdness in the double-slit experiment

The plot thickens, however. In the experiment to determine which slit the electron goes through, one observes a seemingly strange dependence on the frequency of the light used. For $\lambda < d$, we can tell which slit the electron went through, but as a result we destroy the electron interference. For $\lambda > d$, we get the interference back, but we can't tell which slit the electron went through. We explained these two observations with two ideas:

- For $\lambda > d$, the wavelength is too long to tell the difference between the two slits – we can't resolve the slits.
- For $\lambda < d$, we say that shining the light on the system disturbs the system. This is why the interference goes away, and why the result is changed: $P_{12} = P_1 + P_2$ *if* we observe which slit the electron goes through. We check this by turning the light dimmer, and indeed, if we don't see which slit, we get the interference back

These two explanations are correct. However, they open up a new question. For $\lambda > d$, we see interference *no matter how bright the light is.* This seems to contradict the second explanation. One would think that no matter what the wavelength is, if you turned the light bright enough, you would eventually disturb the system. But no such thing happens: you can't destroy the interference as long as $\lambda > d$.

The explanation for this gives us a whole new weird concept: light is a particle. We'll come back to this, but in the next lecture I'l explain another experiment which gives the same conclusion.