

Lecture 34

- General Relativity
- Born, chapter III (most of which should be review for you), chapter VII
- Fowler, “Remarks on General Relativity”
- Ashby on General relativity and the GPS, www.phys.lsu.edu/mog/mog9/node9.html

No absolute acceleration

Everything we did in special relativity involved inertial frames: ones which moved at constant velocities with respect to each other. All this implied that there was some way of distinguishing an accelerating frame from an inertial frame. This, for example, was crucial in understanding the twin paradox. In that case, one can indeed distinguish the two, because the twin in the rocket needs to apply a force (fire the rocket engines) in order to switch inertial frames.

However, when it comes to worrying about forces from gravity, things are subtler. In particular, if we are to distinguish inertial frames from accelerating ones, then this implies the existence of an “absolute rest frame”, a sort of gravitational ether. Born gives a nice example of this. Consider two planets, extremely far from each other, so that the gravity of one can’t possibly affect the other. Say they are both rotating at different speeds around the axis which connects one to the other. Now each will see the other one is an accelerating frame, while I’m at rest.

So who’s right? Well, say one planet is more oblate than the other (fatter at the middle, because of centrifugal forces). Then one might say that the more oblate planet is in the accelerating frame. But this then implies the existence of an absolute rest frame: the more the planet accelerates with respect to the absolute frame, the more oblate it gets. This is the only explanation, because the planets are so far apart that gravity can’t possibly explain the oblateness.

This is logically consistent: one could postulate the existence of an absolute rest frame. If you’re moving at a constant velocity with respect to this, you’re in an inertial frame. If you’re accelerating with respect to this frame, you’re not. The amount of oblateness depends on your acceleration with respect to the absolute rest frame. But since Einstein demolished the idea of an absolute time frame or absolute space frame (i.e. the electromagnetic ether), the thought of an absolute rest frame was fairly suspicious.

Moreover, just like with the electromagnetic ether, there was no experimental evidence for this absolute frame. The reason is that there's no way of just isolating two planets like this. You can look up to the sky and see a frame: that of all the stars. Even though the stars are moving around with respect to each other, it's so far away that we can't really tell over any appreciable time. So they look fixed in space to us. This means we can measure acceleration *relative* to this "fixed" frame. So we don't need the idea of an absolute frame to explain the oblateness, just a relative statement: the more a planet is accelerating with respect to the "fixed" stars, the more oblate it is. As Born points out, since we don't need the idea, why should we assume it?

Junking the idea of an absolute rest frame leads to a huge extension of our principle of relativity: we say now that the laws of physics must involve only relative positions and motions. Thus there is no distinguished set of inertial frames: we can only talk of relative accelerations, not absolute ones. This principle is called *general relativity*.

The Equivalence Principle

This "general" relativity doesn't change anything about electromagnetism. Nothing in there really requires the existence of an absolute rest frame (indeed we got rid of the idea of an ether). But for gravity, this general relativity has profound consequences.

The reason is because of something which always seemed a coincidence, but which in fact turns out to be a fundamental fact of nature. This is the equivalence between inertial and gravitational mass. No one ever points this out to you when you first learn this, but mass appears in two different ways. The first is in $\sum F = ma$ (or in the special-relativistic generalization). The second is in the force due to gravity, either $F_{grav} = mg$ (or the more general GMm/r^2). The fact that for every single body in the universe seems to have the same m in $F = ma$ and $F = mg$ is the equivalence of inertial and gravitational masses.

First let's consider the case of a constant gravitational field. For example, say you stay right at the earth's surface, so $F_{grav} = mg$.) The principle of equivalence means that if there are no other forces in the problem, you can't measure m . This is Galileo's famous experiment: all bodies fall with the same acceleration, independent of mass. But our principle of general relativity means that g is a relative quantity as well: it is the acceleration with respect to the surface of the earth.

So far that doesn't sound so weird. But now say I'm in free fall. Then I don't feel the effect of gravity: if I'm on a scale which is also in free fall, the scale registers zero. So there are two ways of viewing the situation. People on the earth's surface would say that there's a gravitational field, and I'm accelerating because of it. I, on the other hand, would say that there's no gravity, and that for some reason the earth is accelerating toward me. Here's the profound consequence of general relativity: you get the same physics in a uniformly accelerating frame as you do in a

frame without acceleration in the absence of gravity. If I do an experiment (with the lab in free fall too), I get the same results as someone would get in a frame in the middle of outer space, where there is no gravity.

So let me reiterate: a frame in free fall is just like an inertial frame (without gravity)! Observed gravity is the same as observed acceleration. For example, say you're in a closed box resting on some platform. Say I then lower the platform with an acceleration a even greater than that of gravity, so $|a| > g$. But in the box, all you know is that all of a sudden you're *rising* with an acceleration $a - g$. You have no way of telling whether or not I'm pulling the box downward, or a large planet is now above the box, creating a gravitational field which is larger than the earth's.

Because of the principle of equivalence, you can effectively cancel the effect of gravity by going to an accelerating frame. Or you can make it greater, it's up to you. (This is only true for uniform gravity: when the gravitational field depends on space, changing frames only allows you to cancel gravity in any one region where it is constant.) Only because of the principle of equivalence can you do this: because different particles have different charge-to-mass ratios, you can never transform to a frame where all electromagnetism cancels.

This means gravity bends light. I'm in free fall, and shine a flashlight. I'm a perfectly good inertial frame, and say that the light goes in a straight line. On the ground, the light's trajectory will look curved. The only way of understanding the curving with respect to the earth is to note that I see no gravity, but on the ground they do. So in my frame, there's no gravity, light travels in straight lines. On the ground they say there is gravity, so gravity must bend light!

This bending was observed for the first time during a solar eclipse in 1919 by stars very close to the sun. Their light is bent as it passes around the sun. The eclipse doesn't matter for the bending – it's just necessary to see the stars.