Modern Physics notes – Spring 2007
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Lecture 38

- Dark matter and energy
- Cosmic Microwave Background

- Weinberg, chapters II and III
- cosmological parameters: Spergel et al, http://arxiv.org/abs/astro-ph/0603449
- dark matter observations:
  http://www.bell-labs.com/org/physicalsciences/projects/darkmatter/darkmatter1.html

Dark matter and energy

Another question one can ask: is the universe spatially flat at large scales? We know space-time is curved close to large gravitational objects, but at very long distances, does this remain so for space alone? It turns out that the answer to this question is related to the issue of the critical density. It turns out that if the energy density is equal to "critical energy density"

$$\epsilon_c = \rho_c c^2 = \frac{3H^2c^2}{8\pi G} = 8.3 \times 10^{-9} J/m^3$$

then the universe is flat. It is now established to a few percent accuracy that the universe is indeed flat. (This is found by measuring fluctuations in the cosmic microwave background). Thus the energy density of the universe is equal to this energy density.

Now if you try to measure the matter density of the universe, you find two interesting things. First of all, if you measure "visible" matter: things made up of protons, neutrons, electrons, etc, you find that its density is far below that of ρ_c (it's only about a few percent). But the first interesting thing you find (by looking at the effects of gravity) is that there seems to be a lot more matter out there that does not shine, and in fact does not seem to be made up of kinds of matter we know about. There are not enough neutrinos, black holes or Jupiters to account for all the effects of gravity we see. Thus there must be "dark matter" out there. It is one of the great current mysteries of current astrophysics to understand what kind of matter this dark matter is.

So there's all this stuff we can only see by the effects of its gravity. However, the energy in the dark matter and the visible matter is now measured to be have a density ϵ_m obeying

$$\frac{\epsilon_m}{\epsilon_c} = .28 \pm .04$$

This is an incredibly small number: using $E = mc^2$ we can turn this in to a mass density of $2.6 \times 10^{-27}\, kg/m^3$. This is just about 2 protons per m^3!

I told you that $\epsilon = \epsilon_c$, and that matter is only about a third of this. Thus 70% of the energy density of the universe is not in the form of matter: it is "dark energy". It's not the same as dark matter: that's matter of a type we don't know, but which gravitates the way ordinary matter does. The dark energy is called dark because we can't see it, but behaves very differently from ordinary matter (dark or visible). It ends up making massive objects *repel* each other.

The universe is in fact not decelerating, as Newtonian gravity requires. Recent measurements of many supernova have shown that the universe is *accelerating*! Supernova have a useful property like the Cepheid variables: you can use them as standard "candles", because we know their absolute luminosity. Thus if we measure their luminosity on earth, we can tell directly how far they are away. But from the red shift we can tell their velocity. We also know the Hubble constant pretty well now. Putting this all together, they've shown that these supernova are farther than they should be if the universe was decelerating. The simplest explanation for this so far is that the cosmological constant is not zero. The cosmological constant also explains this missing dark energy (we know the universe is spatially flat so that $\epsilon = \epsilon_c$, but from galactic rotation curves we know that there's not enough matter to account for this).

There is a even newer reason why we believe the dark energy is there. This is from something we mentioned before: the light from very old supernova ($z > 1$). You can do the same measurements they did for the supernova before. Then you find that these very old supernova are *closer* than they should be. This means that the universe was *decelerating* before it was accelerating. This is just what we expect. A long time ago, the universe hadn't expanded nearly as much as it is now. That means the galaxies (matter) were much closer together than they are now. Thus gravity was much more dominant than dark energy then, causing the deceleration. But as the universe expanded, the matter got further and further apart, so the attraction of gravity had a proportionally smaller effect. Thus the dark energy took over, causing the current acceleration. Eventually, the dark energy wins out, and almost all the energy in the universe will be in the form of dark energy. Luckily, it seems that it will never be strong enough to rip apart stars, planets, us.

The simplest theoretical explanation of the dark energy is one I mentioned before: Einstein's cosmological constant! But remember that general relativity is a purely classical theory. Thus if indeed the acceleration and the dark energy are described accurately by including this term in Einstein's equations, we still would need to know how quantum mechanics fits. In particular, a question is why the cosmological constant is so small but not zero. Quantum physics does predict a non-zero cosmological constant. Since we don't know how to do quantum gravity, we can't do any reliable computations, but a naive estimate predicts a cosmological constant on the order 10^{124} larger than the one observed. Quantum gravity is not understood!

Cosmic Microwave Background

So we now know we're doomed, the universe's expansion is accelerating, and will continue to do so until the end of time. We now want to go backwards in time, and figure out what happened then. It is now well established that there was a "big bang", an explosion which is responsible for all this matter flying away from each other. The theory of the big bang is basically the subject of Weinberg's book.

One piece of evidence for the big bang is the expansion of the universe – virtually everything is moving away from us. This implies that if you go far enough back in time, the universe was very dense, and some kind of explosion flung everything apart. There's another strong piece of evidence that everything we see was once much closer together. This comes from the fact that there is (very faint) light everywhere we see. No matter which direction or how far we look, one can detect this light. Moreover, the light seems to have almost the identical properties everywhere (i.e. is extremely isotropic and homogeneous), even in parts of the universe which are not at all causally connected any more. This light is called the *cosmic microwave background.*

The only sensible way of interpreting this light is that it has come come from a long long time ago, even longer than these very old supernovas. It is a relic of the very hot universe, coming from not very long after the big bang: only 700,000 years after the big bang (since the universe is 14 billion years old, 99.995% of the time in the universe has happened since the light was emitted). Precise measurements of these microwaves (the latest from a satellite called WMAP) are a major part of the reason why all these cosmological measurements have gotten so good in recent years.

So what is this background? There's a whole chapter in Weinberg devoted to explaining what this precisely means, but we've already studied the physics behind it. It's black-body radiation, where the entire universe is the box!

Remember what this means is that inside a box with photons in thermal equilibrium at temperature T, the energy density $dE(\omega)$ of photons with frequency between ω and $\omega + d\omega$ is

$$dE(\omega) = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}\frac{\omega^2 d\omega}{\pi^2 c^3}$$

The number density of photons with frequency in this range is $dE/(\hbar\omega)$. What the experimentalists measure, now to spectacular accuracy, is that coming from every direction, we are being bombarded with photons with an energy distribution of this form. The temperature of this distribution is $2.735\,K$. The peak of this distribution is in the microwave range, as you will check on the homework.

This observation is a huge piece of evidence for the big bang for several reasons. First of all, the CMB is very isotropic: they measure the same $T = 2.735K$ coming from all directions (after you account for the motion of our galaxy with respect to this background). This means that at some point in the past, these regions were causally related to one another. This implies they all came from the same place, only became causally disconnected as the universe expanded. In fact, the first time any anisotropies were measured was by the satellite COBE 10 years ago. These anisotropies, for reasons I'll explain, are windows into the very early universe.

Second, this implies that the universe was once very hot. Here's the reason. As the universe expands, everything spreads out from each other, even the photons. These photons are no longer interacting with anything (except some minuscule fraction of them hits planets like ours), so no photons are created or destroyed. Thus if the universe is twice the size, the number of photons in a box of fixed size goes down by a factor of $8 = 2^3$. More generally, if the distance increases by a factor of f, the number density of photons decreases by a factor $1/f^3$. The wavelength of these photons also decreases, by the red shift we talked about before. The way to see this is to think about the photons we measure over time. As time goes on, the photons which are hitting us have come from points farther and farther away. Because of the Hubble formula $v = Hd$, points farther and farther away are moving with respect to us at greater and greater velocities and thus are red-shifted more and more. Thus when distances increase by a factor f, the wavelength of the photons we're measuring has increased by a factor f.

This factors of f let us answer the question: if the temperature of the CMB is T now, what was it in the past? Look at the formula for the number density

$$dN(\omega) = \frac{1}{e^{\hbar\omega/k_B T} - 1} \frac{\omega^2 d\omega}{\pi^2 c^3}$$

The left-hand side decreases as $1/f^3$ over time. Because $\omega \to \omega/f$, the right-hand-side does as well, as long as T decreases as T/f as well. This means the universe is cooling! As the universe expands, it cools, just like gases do. The fact that we measure a temperature of $T = 2.7\,K$ now means that the universe must have been very hot in the past. At times just after the big bang, the temperature was incredibly large.

We saw that the total energy density (integrated over all frequencies) of a black body is proportional to T^4. Therefore in the early universe, most of the energy of the universe was in photons, not matter. As the universe cooled off, the energy density ended up mostly in the form of matter, and now it's changing over to dark energy. As the temperature gets lower and lower, the photon energy goes down. The matter will always have its rest mass, no matter how cold it gets. So it ends up dominating the energy density. The transition between the two took place at about 700,000 years after the big bang. This is where the temperature of the universe was about 3000K. Before this, the temperature was hot enough to prevent atoms from forming: if an electron became bound to a nucleus, a photon in the CMB would have enough energy to come along and knock it off. Below 3000K, the photons do not have enough energy to do this any more. Thus as the universe expanded, the photons interacted less and less with the matter, cooling off to form the CMB we see today.

The last paragraph points to another mystery. We still don't know why there were more particles than antiparticles so that when the positrons annihilated, there were still some electrons left. Likewise, in the even earlier universe we don't know why there were more protons than antiprotons and neutrons than antineutrons. If not for this, all the matter would have just annihilated, and just left photons in the universe. To complicated the mystery, the number of photons left is far larger than the number of particles (by a factor of about 10^8).

To see roughly where the 3000K came from, remember that the peak energy of a photon in a black-body distribution obeys

$$E \approx 5k_B T$$

The fact that it $\propto T$ follows from dimensional analysis; the factor of 5 comes by taking the first derivative and solving the resulting equation numerically. Recall the binding energy of hydrogen is about $14\,eV$. Thus when the photons have energy less than $14\,eV$, hydrogen not be knocked apart by the CMB, and thus the two will not interact. The energy of a photon at the peak of the distribution is $14\,eV$ at a temperature

$$\frac{14\,eV}{5k_B} = \frac{(14\,ev)(1.6 \times 10^{-19} J/eV)}{5 \times 1.4 \times 10^{-23} J/K} = 30000K$$

This is the temperature where the peak of the distribution is at $14\,eV$, but at 30000K there are still lots of photons with energy greater than this. Moreover, another fact (not yet explained) is that there are lots more photons than there are particles. So even below 30000K, there are still lots of photons which have enough energy to break apart hydrogen. Thus we have to cool well past 30000K to get rid of these as well. A more detailed computation gives 3000K for the temperature at which