

Lecture 39

- The CMB and the Big Bang
- The First Three Minutes
- Weinberg, chapters III-V (and the mathematical notes in back)

The CMB and the Big Bang

So we've now established to great experimental accuracy that we're in a giant microwave oven at $2.735 K$. In fact, if you work out how many photons there are as compared to protons and neutrons (which together are called *baryons*), the number density of photons is about a billion times that of protons and neutrons. So there's lots of them out there. However, right now the cosmic microwave background is far too cold to interact substantially with matter – the photons don't nearly have enough energy. That's why it took til 1965 and a gigantic dish to even discover the existence of the CMB.

But last time I noted that the CMB used to be a lot hotter – the photons in the CMB cool off as the universe expands. So at what temperature did the photons in the CMB interact with matter in the universe? Remember that the peak energy of a photon in a black-body distribution obeys

$$E \approx 5k_B T$$

The fact that it is $\propto T$ follows from dimensional analysis; the factor of 5 comes by taking the first derivative and solving the resulting equation numerically. Recall the binding energy of hydrogen is about $14 eV$. Thus when the photons have energy less than $14 eV$, hydrogen not be knocked apart by the CMB, and thus the two will not interact. The energy of a photon at the peak of the distribution is $14 eV$ at a temperature

$$\frac{14 eV}{5k_B} = \frac{(14 eV)(1.6 \times 10^{-19} J/eV)}{5 \times 1.4 \times 10^{-23} J/K} = 30000K$$

As I said above, there are lots more photons than there are particles. At 30000K, the more energetic ones in the distribution have enough energy to break apart hydrogen, and there are lots of them. Thus we have to cool well past 30000K to get rid of these as well. A more detailed computation gives 3000K for this.

So when the temperature of the CMB was greater than 3000K, it prevented atoms from forming – if one formed, a photon would come along and knock the electron out. Before this

time, the universe was *opaque*: the radiation couldn't get through without scattering off of matter. Thus as the universe expanded, the photons interacted less and less with the matter, cooling off to form the CMB we see today, which barely interacts at all. This is why it's a window on the early universe – except for cooling, it's the same as it was when the entire universe was 3000 degrees! The photons we're measuring now were emitted then.

By measuring the CMB, we're doing experiments on the early universe. One thing you find is that the CMB is very isotropic. No matter which direction you look, the CMB looks almost the same: you get the same 2.735 K looking up and looking down. This is even though these regions are now *causally disconnected*: there's no way a photon coming from above now knows anything about any photon coming from below – there's no way to get a message from one photon to another without violating the speed limit of c . The fact that they both know they're at 2.735 K means that at some time in the past they were much closer together. This is substantial evidence for the Big Bang!

In the last 15 years it has become possible to measure *anisotropies* in the CMB. They're very small, but are there. These are the best experimental way we have of telling what's going on the early universe. Whatever your model of the early universe is had better give anisotropies in the CMB which agree with what we see. This was how it was determined that the universe is flat: if it weren't the CMB photons would have been bent by this, and we don't see it.

The time line

We now have seen that the universe used to be a lot hotter and more dense. We can use the information we have to construct a time line for the universe. From now on, we're going to neglect the acceleration from the dark energy. We can do this because the early universe is much more dense than the universe now, so gravity then dominates the dark energy.

So now consider a bunch of matter in some large region of the universe of radius $R(t)$. I made this depend on time, because we're going to follow the matter and energy in this region as the universe expands. We saw before that the energy of a galaxy of mass m at the edge of this

$$E = mR^2(t) \left[\frac{1}{2}H^2(t) - \frac{4}{3}\pi\rho(t)G \right]$$

where $\rho(t)$ is the density of matter in this region. The first term came from kinetic energy, the second from gravity. Now by energy conservation, this energy has to stay the same as the universe expands. This means that the term in brackets must essentially vanish. (Precisely, it must go as $1/R^2(t)$, which gets smaller and smaller as the universe expands.) The vanishing means that

$$\frac{1}{2}H^2(t) \approx \frac{4}{3}\pi\rho(t)G$$

Note that the Hubble constant is a constant in *space*, not in time. This I hope is obvious: as the universe expands, the distance between galaxies increases, but if there is no force, the velocity doesn't change. Thus H must change for $v = Hd$ to hold. Moreover, the gravitational attraction between matter can (and does) cause it to change as well.

We can use this to develop a timeline for the universe. The density $\rho(t)$ depends on what kind of matter it is. First, remember we showed that the average distance between galaxies is doubled, then the temperature is halved. In an equation,

$$R(t) \propto \frac{1}{T(t)}$$

For ordinary matter in our region of size $R(t)$, as the universe expands, the same amount of energy takes up a larger space. For matter, we can assume that the energy is all in the form of its rest mass, so

$$\rho_{\text{matter}}(t) \frac{4\pi}{3} R^3(t) = \text{constant}$$

Thus

$$\rho_{\text{matter}}(t) \propto R^{-3}(t) \propto T^3(t)$$

For photons, it's different, since of course they have no rest energy. By taking the integral of the black-body energy formula over all frequencies, you get the total energy density. We saw that

$$\rho_{\text{photons}} \propto T^4$$

We thus have

$$\rho(t) \propto T^n$$

where $n = 3$ if the energy in the universe is dominated by ordinary matter, and $n = 4$ if it is dominated by photons.

So now we have enough information to relate time to density. We have

$$H(t) \propto \sqrt{\rho(t)} \propto T^{n/2} = A[R(t)]^{-n/2}$$

for some time-independent constant A . We thus have for the relative velocity $v(t)$ of galaxies a distance $R(t)$ apart:

$$v(t) = H(t)R(t) = A[R(t)]^{1-n/2}$$

But of course $v(t) = dR(t)/dt$. We thus have

$$\frac{dR(t)}{dt} = A[R(t)]^{1-n/2}$$

The trick solve this is to write it as

$$R^{n/2-1}dR = Adt$$

Integrating both sides from t_1 to t_2 gives

$$\frac{2}{nA}(R^{n/2}(t_2) - R^{n/2}(t_1)) = t_2 - t_1$$

Thus we know can make a timeline: we've related the time elapsed to the change in radius. We can eliminate the unknown constant A by using the fact that $H(t) = AR(t)^{-n/2}$.

$$t_2 - t_1 = \frac{2}{n} \left[\frac{1}{H(t_2)} - \frac{1}{H(t_1)} \right]$$

Since we related the Hubble constant to the density, we have

$$t_2 - t_1 = \frac{2}{n} \sqrt{\frac{3}{8\pi G}} \left[\frac{1}{\sqrt{\rho(t_2)}} - \frac{1}{\sqrt{\rho(t_1)}} \right]$$

So finally, we can find out how long it took the universe to cool to 3000 degrees, the temperature at which the CMB stopped interacting with matter. So let t_2 be this time. Let t_1 be some time just after the big bang (say just a few seconds). Here the universe is very dense, so $\rho(t_1) \gg \rho(t_2)$. Our formula then gives

$$t_{3000} = \frac{2}{4} \sqrt{\frac{3}{8\pi G\rho(t_{3000})}}$$

I've used $n = 4$ because at this point, the energy of the universe is dominated by photons, because there are so many more of them (remember their energy goes as T^4 , which beats out the T^3 when the universe is hot). From the CMB data, we know what the density of photons were at t_{3000} ; it is

$$\rho(t_{3000}) = 10^{-21} g/cm^3$$

Plugging this in gives

$$t_{3000} = 700,000 \text{ years}$$

So the CMB is a window on the universe just 700000 years after the big bang. Since the universe is now 14 billion years old, only .05% of the time had elapsed when this happened. Those photons hitting you when you walk outside were created 14 billion years ago!

The first three minutes

We have seen that in the early universe, most of the energy of the universe was in photons, not matter. As the universe cooled off, the energy density ended up mostly in the form of matter, and now it's changing over to dark energy. The story of our universe is the story of this conversion of energy into the matter which makes up our world. Since the data really only goes back to the CMB, going back any further in time (or higher in temperature) requires using

theory, and then comparing the consequences at the very early times to the CMB data. But we now have a pretty good picture of what happened, although there are of course many mysteries.

It's basically a story of binding energy. When the universe has cooled enough for matter to become stable, it does. Nuclear binding energies are much higher than atomic ones: this is why nuclear bombs are much more powerful than chemical ones. Thus nuclei form at a much higher temperature: this happens at about $10^9 K$, at a time about 3 minutes and 46 seconds after the big bang. This is called "nucleosynthesis", and it is understood in very precise detail: we know accurately the ratio of hydrogen to helium (or equivalently, the ratio of protons to neutrons) in the universe. (It's about 26 % helium by weight.)

At earlier times, it's a soup of electrons, positrons, neutrinos, with much smaller amounts of protons and neutrons, and still smaller amounts of antiprotons and antineutrons. As the universe cools, the antiparticles annihilate with their partners to produce photons. It's still a mystery why there were more particles than antiparticles so that when the positrons annihilated, there were still some electrons left. Likewise, in the even earlier universe we don't know why there were more protons than antiprotons and neutrons than antineutrons. If not for this, all the matter would have just annihilated, and just left photons in the universe. To complicate the mystery, the number of photons left is far larger than the number of particles (by a factor of about 10^8).

At even earlier times, it's quark soup. At even earlier times, we're not sure what's going on. In order to explain isotropy of the CMB, and the flatness of the universe, a strong possibility is that at very small times (something like 10^{-35} seconds!) the universe underwent a very rapid expansion, called inflation. Inflation predicts very specific kinds of anisotropy in the CMB, and so far the data is consistent. But even if this is confirmed, there are many possibilities for what caused the inflation to happen, so there still will be many mysteries!